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PARTIAL SUMS OF CERTAIN ANALYTIC FUNCTIONS

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Abstract. Let $f_n(z) = z + \sum_{k=2}^{n} a_k z^k$ be the sequence of partial sums of the analytic function $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$. We determine sharp lower bounds for $\operatorname{Re}\{f(z)/f_n(z)\}$, $\operatorname{Re}\{f_n(z)/f(z)\}$, $\operatorname{Re}\{f'(z)/f_n'(z)\}$ and $\operatorname{Re}\left\{f_n'(z)/f^{'}(z)\right\}$ under certain conditions.

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