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Φ-LIKE FUNCTIONS IN TWO-DIMENSIONAL FREE BOUNDARY PROBLEMS

PAULA CURT, DENISA FERICEAN, and TEODOR GROŞAN

Abstract. In this paper we apply certain results in the theory of univalent functions to investigate the time evolution of the free boundary of a viscous fluid for a planar flow problem in the Hele-Shaw cell model under injection. More precisely, we prove that the property of strongly Φ-likeness of order $\alpha \in (0, 1]$ (a geometric property which includes strongly starlikeness of order α and strongly spirallikeness of order α) remains invariant in time for two basic cases: the inner problem and the outer problem, under the assumption of zero surface tension. Special cases that are obtained by using numerical computations are also presented.

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Key words. Conformal map, free boundary problem, Hele-Shaw flow, Φ-like function, spirallike function, starlike function, strongly Φ-like function.

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"Babeş-Bolyai" University
Faculty of Economics
and Business Administration
Cluj-Napoca, Romania
E-mail: paula.curt@econ.ubbcluj.ro

"Babeş-Bolyai" University
Faculty of Mathematics and Computer Science
Cluj-Napoca, Romania
E-mail: denisa.fericean@ubbcluj.ro
E-mail: tgrosan@math.ubbcluj.ro