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# BESSEL TRANSFORMS AND HARDY SPACE OF GENERALIZED BESSEL FUNCTIONS

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**Abstract.** In this paper, which was motivated by the papers of S. Ponnusamy [11, 12], we continue the study of generalized and normalized Bessel functions of the first kind of real order. We present some immediate applications of convexity and univalence involving Bessel functions associated with the Hardy space of analytic functions, i.e. we obtain conditions for the function

$$u_p(z) = \sum_{n=0}^{\infty} \left(-\frac{c}{4}\right)^n \frac{\Gamma\left(p + \frac{b+1}{2}\right)}{\Gamma\left(p+n+\frac{b+1}{2}\right)} \frac{z^n}{n!},\ b,p,c \in \mathbb{R}, z \in \mathbb{C}$$

to belong to the Hardy space $\mathcal{H}^\infty$. Let consider $\mathcal{A}$, the class of all analytic and normalized functions in the unit disk and

$$\mathcal{R}(\alpha) = \{f \in \mathcal{A} : \exists\ \eta \in \mathbb{R} \text{ such that } \operatorname{Re}\ [\mathrm{e}^{\mathrm{i}\eta}(f'(z) - \alpha)] > 0, z \in U\}.$$

When $\eta = 0$ we denote $\mathcal{R}(\alpha)$ simply by $\mathcal{R}_0(\alpha)$, and when $\alpha = 0$, we denote $\mathcal{R}_0(\alpha)$ simply by $\mathcal{R}$. We find conditions for the convolution $zu_p(z) * f(z)$ to belong to $\mathcal{H}^\infty \cap \mathcal{R}$, where $f$ is an analytic function in $\mathcal{R}$. Finally we obtain conditions for $\alpha_1, \alpha_2$ and the parameters $b, c, p$ such that the operator $B(f) := zu_p(z) * f(z)$ maps $\mathcal{R}(\alpha_1)$ into $\mathcal{R}(\alpha_2)$.

**MSC 2000.** 30C45, 30A20.

**Key words.** Analytic functions, univalent functions, starlike functions, convex functions, Bessel functions, Hardy space.

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