

GEOMETRIC PROPERTIES OF GENERALIZED
BESSEL FUNCTIONS OF COMPLEX ORDER

ÁRPÁD BARICZ

Abstract. In this paper we obtain conditions of univalence and convexity for the generalized and normalized Bessel functions of the first kind of complex order using the technique of differential subordinations. A condition of starlikeness of $zu_p(z)$ is given, where by definition

$$u_p(z) := \sum_{n=0}^{\infty} \left(-\frac{c}{4}\right)^n \frac{\Gamma\left(p + \frac{b+1}{2}\right)}{\Gamma\left(p + n + \frac{b+1}{2}\right)} \frac{z^n}{n!}, \quad b, p, c, z \in \mathbb{C}.$$

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Key words. Differential subordination, convex functions, univalent functions, starlike functions, Bessel functions.

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*Faculty of Mathematics and Computer Science
“Babeş-Bolyai” University
Str. M. Kogălniceanu nr. 1
400084 Cluj-Napoca, Romania
E-mail: bariczocsi@yahoo.com*

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