

ON A CONJECTURE OF LIVINGSTON

FARIT G. AVKHADIEV and KARL J. WIRTHS

**Abstract.** Let  $D$  denote the open unit disc and  $f : D \rightarrow \overline{\mathbf{C}}$  be meromorphic and injective in  $D$ . We further assume that  $f$  has a simple pole in the point  $p \in (0, 1)$  and an expansion

$$f(z) = z + \sum_{n=2}^{\infty} a_n(f)z^n, \quad |z| < p.$$

Especially, we consider  $f$  that map  $D$  onto a domain whose complement with respect to  $\overline{\mathbf{C}}$  is convex. Concerning a (sharper) conjecture of Livingston ([5]) we prove that for  $n \geq 2$  the inequality

$$\operatorname{Re}(a_n(f)) \geq \frac{1 + p^{2n}}{p^{n-1}(1 + p)^2}$$

is valid.

**MSC 2000.** 30C45, 30C50.

**Key words.** Taylor coefficients, concave univalent functions, starlike meromorphic functions.

The authors thank Ch. Pommerenke for helpful discussions and the Deutsche Forschungsgemeinschaft for support during research on papers related to the present one.

REFERENCES

- [1] AVKHADIEV, F.G. and WIRTHS, K.-J., *Convex holes produce lower bounds for coefficients*, Compl. Var., **47** (2002), 553–563.
- [2] AVKHADIEV, F.G. and WIRTHS, K.-J., *Poles near the origin produce lower bounds for coefficients of meromorphic univalent functions*, Mich. Math. J., to appear.
- [3] AVKHADIEV, F.G., POMMERENKE, CH. and WIRTHS, K.-J., *On the coefficients of concave univalent functions*, Math. Nachr., to appear.
- [4] GOLUZIN, G.M., *Geometric theory of functions of a complex variable*, Translations of mathematical monographs, **26**, AMS, Providence, 1969.
- [5] LIVINGSTON, A.E., *Convex meromorphic mappings*, Ann. Pol. Math., **59.3** (1994), 275–291.
- [6] MILLER, J., *Convex and starlike meromorphic functions*, Proc. Amer. Math. Soc., **80** (1980), 607–613.

Received May 9, 2003

*Chebotarev Research Institute*  
*Kazan State University*  
*420008 Kazan, Russia*  
*E-mail: nina@dionis.kfti.kcn.ru*

*Institut für Analysis*  
*TU Braunschweig*  
*38106 Braunschweig, Germany*  
*E-mail: kjwirths@tu-bs.de*