

Report

on the activities associated in 2013 to the project
PN-II-RU-TE-2011-3-0065 (contract 47/05.10.2011)

Functors on Module Categories

Budget: 46.899,68 lei

Activities

The activities were developed as in the initial project in the following way:

In order to conform the knowledge and the language for all members of our team we organized seminars (presented by Claudiu Vâlculescu). During these seminars we studied basic notions of homological algebra, following the course notes *Notes on Homological Algebra* by Jeke Moerdijk.

(I) Functors on module categories

We identified functors which are useful in the study of various module categories, and we studied various commuting properties for these functors.

The main research subject in this direction was the study of some commuting properties associated to the functors Hom and Ext , respectively to obtain various correspondences which are induced by these functors:

- (1) universal properties which can be expressed by commuting properties of Hom -functors.
- (2) commuting of covariant Hom functors with respect to direct sums (small and self-small modules);
- (3) commuting of covariant Hom functors with respect to direct various direct limits (finitely generated and finitely presented modules);
- (4) commuting of contravariant Hom functors with respect to direct products (slender and self-slender modules);
- (5) commuting of covariant Ext functors with respect to direct sums;
- (6) finiteness properties which can be expressed by some commuting properties of covariant Ext functors (resultats proved by K.S. Brown si R. Strebel)

The paper:

1. U. Albrecht, **S. Breaz**, P. Schultz, *Functorial properties of Hom and Ext*,

was accepted by *Contemporary Mathematics* (editors L. Fuchs și L. Strümann).

(II) Torsion and cotorsion classes

We studied various aspects concerning perpendicular classes and connections with closure properties with respect some categorial constructions:

- (1) Closure properties for cotorsion and cotorsion-free classes with respect various categorial constructions: colimits (filtered), products, coproducts etc;
- (2) metods which are useful in order to prove that a cotorsion theory is complete;
- (3) we tried to identify and to study examples of cotorsion pairs;
- (4) torsion and cotorsion pairs induced by a tilting module;
- (5) flat covers for complexes of modules;
- (6) connections between flat covers and the localization induced by the complexes constructed using flat modules in the homotopy category of all complexes associated to a module category.