Positive Series

Determine whether the series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{e^n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{(n+1)^3}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n3^n}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[6]{n^4}}$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n(n+1)(n+2)}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{(2n+1)!}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n+5}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n!}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{\pi}{(2n+3)(n-5)}$

15. $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 2^n}{(2n)!}$

16. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+n}}{n^3}$

17. $\displaystyle\sum_{n=2}^{\infty} \frac{n(1+n^2)}{n^3-3}$

18. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n 3^n}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3+2^{-n}}$

20. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n[\ln n]^{3/2}}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n+2^n}{4^n}$

22. $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^8+6n^3}}$

24. $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n+2}\right)$

25. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{\ln n} + \frac{3}{4^n}\right)$

26. $\displaystyle\sum_{n=1}^{\infty} (e^{1/n} - 1)$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+1}{n!}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{5n}{(n^7+3)^{1/3}}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{n(n-1)}{(n+1)(n+2)}$

31. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{n}{\ln n} - \frac{5}{2^n}\right)$

32. $\displaystyle\sum_{n=1}^{\infty} \frac{5n-6}{3n^{5/2}+2n-2}$

33. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{2^n n^2}$

34. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2^n+n}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{e^n}$

38. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n(n+3)}$

41. $\displaystyle\sum_{n=1}^{\infty} \frac{3n^4-2n^3+7}{5n^7+n^2-3n}$

42. $\displaystyle\sum_{n=2}^{\infty} \frac{n+3}{(\ln n)^2}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{\sqrt{n^5+2}}$

44. $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2}{(n+1)(n+2)(n+3)(n+4)}$

45. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)^{1/2}}$

46. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\ln^2 n}$

47. $\displaystyle\sum_{n=1}^{\infty} \sin n\pi$

48. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^3 - 1000}$

49. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n \ln n}$

50. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n + 1}$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

52. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n + 1)}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n + 2}}{n^3 + 1}$

54. $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 1)!}{(n - 1)! 2^n}$

55. $\displaystyle\sum_{n=1}^{\infty} \left(1 + \frac{4}{n}\right)^n$

56. $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n + 1}$

57. $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{10n^3 + 7n^2}{n^{3/2} + 3}$

59. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$

60. $\displaystyle\sum_{n=1}^{\infty} \frac{7^n}{(3n)!}$

61. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3 \cdot 2^n + 5}$

62. $\displaystyle\sum_{n=1}^{\infty} \frac{n + 2}{n\sqrt{5n - 2}}$

63. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 2n + 1}{(n - 3)(n + 2)}$

64. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{99 + n^2}}$

65. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2 + \frac{1}{n})^n}$

66. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

67. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

68. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 3n + 1}{4n^3 + 1}$

69. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{3/2}}$

70. $\displaystyle\sum_{n=2}^{\infty} \frac{4}{3^n}$

71. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$

72. $\displaystyle\sum_{n=1}^{\infty} \ln \left(\frac{n + 1}{2n + 1}\right)$

73. $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k!}$

74. $\displaystyle\sum_{n=1}^{\infty} \frac{5n}{(n^7 + 3)^{1/3}}$

75. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \sqrt[3]{\ln k}}$

76. $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^2 + 4}$

77. $\displaystyle\sum_{m=1}^{\infty} \frac{\arctan m}{1 + m^2}$

78. $\displaystyle\sum_{n=1}^{\infty} \frac{n + 3}{2^n}$

79. $\displaystyle\sum_{n=1}^{\infty} \frac{n + 2}{\sqrt{3n^3 + n + 1}}$

80. $\displaystyle\sum_{n=1}^{\infty} \left\{\frac{1}{2} + \frac{2}{3n}\right\}^n$

81. $\displaystyle\sum_{n=2}^{\infty} \left(\frac{\sqrt{n}}{\ln n} + (-1)^n \frac{1}{3^n}\right)$

82. $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 2)!}{4^{n+1}}$

83. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$

84. $\displaystyle\sum_{n=1}^{\infty} \frac{5\sqrt{n^2 + 2}}{3n + 1}$

85. $\displaystyle\sum_{n=1}^{\infty} \frac{2\sqrt{n} + 5n}{\sqrt{3n^3 + 4}}$

86. $\displaystyle\sum_{n=3}^{\infty} \frac{e}{(3n + 2)(2n - 5)}$

87. $\displaystyle\sum_{m=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2m - 1)}{m!}$

88. $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$

89. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos \frac{1}{n}}{\arctan n}$

90. $\displaystyle\sum_{n=1}^{\infty} \frac{3n + 2}{(n + 1)(4n)}$

91. $\displaystyle\sum_{n=1}^{\infty} \ln \left[\frac{n^2 + 2}{3n^2 + 5}\right]$

92. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{(2n + 1)!}$

93. $\displaystyle\sum_{n=1}^{\infty} \left[\frac{n + 3}{2n + 1}\right]^n$

94. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2 + 5^n}$

95. $\displaystyle\sum_{n=1}^{\infty} n e^{-n}$

96. $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)$

97. $\displaystyle\sum_{n=1}^{\infty} \frac{(n + 1)!}{3^{2n+1}}$

98. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} + \frac{1}{5^n}\right)$

99. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{3^n}$

100. $\displaystyle\sum_{n=1}^{\infty} \frac{3^{2n}}{1 + \ln n}$

101. $\sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n^2+3}}$

102. $\sum_{n=1}^{\infty} \sin\left(\dfrac{1}{n}\right)$

103. $\sum_{n=1}^{\infty} \dfrac{4\sqrt{n}-1}{n^2+2\sqrt{n}}$

104. $\sum_{n=1}^{\infty} \ln\left[\dfrac{2n+1}{n}\right]$

105. $\sum_{n=1}^{\infty} \dfrac{3^n}{n5^n+2}$

106. $\sum_{n=1}^{\infty} n\sin\left(\dfrac{1}{n}\right)$

Answers

1. Converges (LCT)

2. Diverges (p-series)

3. Diverges (LCT)

4. Converges (Ratio Test)

5. Converges (LCT)

6. Converges (Ratio Test)

7. Diverges (p-series)

8. Converges (LCT)

9. Converges (Ratio Test)

10. Converges (Ratio Test)

11. Diverges (DT)

12. Diverges (IT)

13. Converges (Ratio Test)

14. Converges (LCT)

15. Converges (Ratio Test)

16. Converges (LCT)

17. Diverges (DT)

18. Diverges (Ratio Test)

19. Diverges (DT)

20. Converges (IT)

21. Diverges (geometric series)

22. Diverges (DT)

23. Converges (LCT)

24. Diverges ($\lim S_n$)

25. Diverges (DT)

26. Diverges (LCT)

27. Converges (IT)

28. Converges (Ratio Test)

29. Converges (LCT)

30. Diverges (DT)

31. Diverges (DT)

32. Converges (LCT)

33. Diverges (Ratio Test)

34. Diverges (IT)

35. Converges (Ratio Test)

36. Converges (CT)

37. Converges (Ratio Test)

38. Converges (IT)

39. Converges (IT)

40. Diverges (Ratio Test)

41. Converges (LCT)

42. Diverges (DT)

43. Converges (LCT)

44. Converges (LCT)

45. Diverges (LCT)

46. Converges (IT)

47. Converges ($\lim S_n$)

48. Diverges (LCT)

49. Diverges (IT)

50. Diverges (DT)

51. Converges (Ratio Test)

52. Converges (LCT)

53. Converges (LCT)

54. Converges (Ratio Test)

55. Diverges (DT)

56. Diverges ($\lim S_n$)

57. Converges (IT)

58. Diverges (DT)

59. Converges (Ratio Test)

60. Converges (Ratio Test)

61. Converges (CT)

62. Diverges (LCT)

63. Diverges (DT)

64. Diverges (LCT)

65. Converges (CT)

66. Diverges (Ratio Test)

67. Converges (Ratio Test)

68. Diverges (LCT)

69. Converges (IT)

70. Converges (geometric-series)

71. Converges (Ratio Test)

72. Diverges (DT)

73. Converges (Ratio Test)

74. Converges (LCT)

75. Diverges (IT)

76. Converges (LCT)

77. Converges (IT)

78. Converges (Ratio Test)

79. Diverges (LCT)

80. Converges (Root Test)

81. Diverges (DT + geometric series)

3

82. Diverges (Ratio Test)

83. Converges (IT)

84. Diverges (DT)

85. Diverges (LCT)

86. Converges (LCT)

87. Diverges (LCT)

88. Converges (IT)

89. Diverges (DT)

90. Diverges (LCT)

91. Diverges (DT)

92. Converges (Ratio Test)

93. Converges (Root Test)

94. Converges (CT)

95. Converges (Ratio Test)

96. Diverges (DT)

97. Diverges (Ratio Test)

98. Diverges (p-series + geom. series)

99. Diverges (Ratio Test)

100. Diverges (DT)

101. Converges (LCT)

102. Diverges (LCT)

103. Converges (CT)

104. Diverges (DT)

105. Converges (CT)

106. Diverges (DT)

LCT = Limit Comparison Test
DT = Divergence Test (the general term does not converge to 0)
CT = Comparison Test
IT = Integral Test (If $f : [1, \infty) \to [0, \infty)$ is decreasing, $\sum_{n=1}^{\infty} f(n) < \infty \iff \int_1^{\infty} f(x)\mathrm{d}x < \infty$)