Sequences, Geometric and Telescoping Series

1. Given the sequence $\left\{\dfrac{1+\ln n}{n^3}\right\}_{n=1}^{\infty}$
   (a) Is it monotonic? Is it bounded?
   (b) What can be concluded from (a)?

2. Illustrate each of the following with an example.
   (a) A bounded sequence need not converge.
   (b) A monotonic sequence need not be bounded.

3. If possible, state an example for each of the following
   (a) A convergent sequence which is not monotonic.
   (b) A convergent sequence which is not bounded.

4. Determine whether each sequence is convergent or divergent? If convergent find what it converges to. If divergent, state when it diverges to ∞ or $-\infty$.
   (a) $\left\{\frac{\sqrt{n}+1}{n}\right\}_{n=1}^{\infty}$
   (b) $\left\{\frac{n^2}{n!}\right\}_{n=0}^{\infty}$
   (c) $\left\{\frac{n!2^n}{(2n)!}\right\}_{n=0}^{\infty}$
   (d) $a_n = -2 + \ln\left[\frac{2+n}{3n}\right], \quad n = 1, 2, 3, ...$
   (e) $a_n = (5n)^{3/\ln n}, \quad n = 2, 3, 4, ...$
   (f) $\left\{\frac{2n+1}{3n-1}\right\}_{n=1}^{\infty}$
   (g) $a_n = \ln(2n+1) - \ln n, \quad n = 1, 2, 3, ...$
   (h) $\left\{\frac{n}{e^n}\right\}_{n=1}^{\infty}$
   (i) $\left\{\frac{2^n}{n!}\right\}_{n=1}^{\infty}$
   (j) $\{1-(-1)^n\}_{n=1}^{\infty}$
   (k) $\left\{\frac{\ln(n+1)}{(n+1)^2}\right\}_{n=1}^{\infty}$
   (l) $\{e^{-n}\sin n\}_{n=1}^{\infty}$
   (m) $\left\{\frac{n^3+2n}{n^2+7}\right\}_{n=1}^{\infty}$
   (n) $\left\{\frac{n+1}{2^n}\right\}_{n=1}^{\infty}$
   (o) $\left\{\frac{3^{n+2}}{(n+1)!}\right\}_{n=1}^{\infty}$
   (p) $a_n = \frac{n}{n^2+n+2}, \quad n = 3, 4, 5, ...$
   (q) $\left\{\frac{e^n}{n!}\right\}_{n=1}^{\infty}$
   (r) $\left\{\frac{\sqrt{n}}{n-3}\right\}_{n=4}^{\infty}$
   (s) $\left\{\frac{2n^2+1}{5n^2-3}\right\}_{n=1}^{\infty}$

5. For each geometric sequence determine its common ratio r, whether it converges or diverges, and find its sum when it converges.
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{4}{(-3)^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{2^{n+2}}$
   (c) $\frac{8}{3} + \frac{64}{27} + \frac{512}{243} + \cdots$
   (d) $1 - e + e^2 - e^3 + \cdots$

6. Determine whether the telescoping sum converges or diverges. Find its sum when it converges.
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(2n+1)(2n+3)}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+3)(n+5)}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$

Answers:

1. (a) Yes. Yes. (b) It converges.
2. (a) $\{(-1)^n\}$ is bounded and oscillating.
   (b) $\{n\}$ is monotonic and unbounded.
3. (a) $a_n = \frac{(-1)^n}{n}$
   (b) Not possible.

   Answers:

4. (a) Converges to 0.
   (b) Converges to 0.
   (c) Converges to 0.
   (d) Converges to $-2 - \ln 3$.
   (e) Converges to e^3
   (f) Converges to $\frac{2}{3}$.

(g) Converges to $\ln 2$.

(h) Converges to 0.

(i) Converges to 0.

(j) Diverges.

(k) Converges to 0.

(l) Converges to 0.

(m) Diverges to ∞.

(n) Converges to 0.

(o) Converges to 0.

(p) Converges to 0.

(q) Converges to 0.

(r) Converges to 0.

(s) Converges to $\frac{2}{5}$.

5. (a) $r = -\frac{1}{3}$. Converges to $\frac{1}{3}$

   (b) $r = \frac{3}{2}$. Diverges to ∞.

   (c) $r = \frac{8}{9}$. Converges to 24.

   (d) $r = -e$. Diverges.

6. Determine whether the telescoping sum converges or diverges. Find its sum when it diverges.

   (a) Converges to

   (b) Converges to

   (c) Diverges to ∞.