Integration

Evaluate each integral.

1. $\int \dfrac{x}{\sqrt{x^2-1}}\, dx$

2. $\int x \cos ax\, dx$

3. $\int \dfrac{7x+1}{x^2-x-2}\, dx$

4. $\int_0^{\pi/2} \cos^3 x\, dx$

5. $\int \dfrac{x^3\, dx}{\sqrt{x+1}}$

6. $\int_0^{\infty} \dfrac{dx}{(x+1)^2}$

7. $\int_1^5 \dfrac{x^2+1}{\sqrt{x-1}}\, dx$

8. $\int \sin^6 x \cos^3 x\, dx$

9. $\int \sin\theta \ln(\cos\theta)\, d\theta$

10. $\int_1^2 \dfrac{dx}{\sqrt{x-1}}$

11. $\int_2^{\infty} \dfrac{dx}{(x-1)^2}$

12. $\int \sec^3 2\theta \tan^5 2\theta\, d\theta$

13. $\int_1^2 x^2\sqrt{2-x}\, dx$

14. $\int \dfrac{dx}{x(x+1)^2}$

15. $\int_0^{\sqrt{3}} \dfrac{dx}{(4-x^2)^{3/2}}$

16. $\int_{-2}^2 \dfrac{dx}{(x+2)^2}$

17. $\int \dfrac{1}{x(x+2)}\, dx$

18. $\int \arctan\sqrt{x}\, dx$

19. $\int_1^{\infty} e^{-x}\, dx$

20. $\int \dfrac{x^3}{\sqrt{4-x^2}}\, dx$

21. $\int_0^1 \dfrac{dx}{x}$

22. $\int_1^5 (x+1)\sqrt{2x-1}\, dx$

23. $\int \sin^5 2\theta \cos^3 2\theta\, d\theta$

24. $\int_0^{\sqrt{2}} \dfrac{x^3}{\sqrt{4-x^2}}\, dx$

25. $\int x \arctan x\, dx$

26. $\int_0^3 \dfrac{dx}{(x-1)^2}$

27. $\int e^x \sin x\, dx$

28. $\int x^2\sqrt{x+1}\, dx$

29. $\int x \sin^3 x^2 \cos^4 x^2\, dx$

30. $\int \cot^3 2x \csc^3 2x\, dx$

31. $\int_{-1}^2 \dfrac{dx}{x^2}$

32. $\int_1^{\infty} \dfrac{dx}{(2x-1)^{3/2}}$

33. $\int 4\cos^3(\tfrac{x}{3}) \sin^2(\tfrac{x}{3})\, dx$

34. $\int_0^{\pi/6} 4\sin^2(3x)\, dx$

35. $\int \sec^2 x \ln|\cot x|\, dx$

36. $\int \sec^4 x \tan^3 x\, dx$

37. $\int \dfrac{x^2}{(9-4x^2)^{3/2}}\, dx$

38. $\int_6^{\infty} \dfrac{dx}{x\sqrt{x^2-9}}$

39. $\int_0^5 \dfrac{dx}{x^2-4x+4}$

40. $\int \sin^2 x \cos^2 x\, dx$

41. $\int \cot^3 x\, dx$

42. $\int \sec^6 x \tan^3 x\, dx$

43. $\int_0^{\pi/4} \sin x \ln(\sec x)\, dx$

44. $\int \dfrac{\sqrt{9x^2-4}}{x}\, dx$

45. $\int_{2\sqrt{3}}^{\infty} \dfrac{dx}{x^2+4}$

46. $\int_0^1 \ln x\, dx$

47. $\int (\tan 2x)^2 \sec 2x\, dx$

48. $\int \dfrac{dx}{x^2\sqrt{x^2-1}}$

49. $\int x^2(\ln x)^2\, dx$

50. $\int_0^{\pi/4} \dfrac{\sin^3 x}{\cos^4 x}\, dx$

51. $\int_0^{\infty} \dfrac{1}{\sqrt{e^x}}\, dx$

52. $\int_0^4 \dfrac{1}{(x-2)^2}\, dx$

53. $\int \sin^3 x \cos^4 x\, dx$

54. $\int \dfrac{dx}{e^x+e^{-x}}$

55. $\int xe^{-x}\, dx$

56. $\displaystyle\int \cos^2(\ln x)\frac{dx}{x}$

57. $\displaystyle\int e^{2x}\cos 3x\, dx$

58. $\displaystyle\int x^3\sqrt{x^2-4}\, dx$

59. $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{4+x^2}$

60. $\displaystyle\int_0^2 \frac{dx}{(x-1)^2}\, dx$

61. $\displaystyle\int \frac{\sec^2 3x}{\tan 3x}\, dx$

62. $\displaystyle\int \frac{x^3}{\sqrt{4x^2-9}}\, dx$

63. $\displaystyle\int_{-2}^2 \frac{dx}{x^3}$

64. $\displaystyle\int_0^{\infty} xe^{-x}\, dx$

65. $\displaystyle\int \frac{5x^2-11x+5}{(x-1)^2(x-2)}\, dx$

66. $\displaystyle\int \ln^2 x\, dx$

67. $\displaystyle\int \frac{x^5}{\sqrt[3]{x^3+1}}\, dx$

68. $\displaystyle\int \sin^3 2\theta\, \cos 2\theta\, d\theta$

69. $\displaystyle\int_{-1}^1 \frac{(\ln|x|)^3}{x}\, dx$

70. $\displaystyle\int_{\sqrt{5}}^{\sqrt{20}} \frac{dx}{(x^2-4)^{3/2}}$

71. $\displaystyle\int_{-\infty}^0 xe^{2x}\, dx$

72. $\displaystyle\int x\tan^2 x\, dx$

73. $\displaystyle\int \tan^4 x\, dx$

74. $\displaystyle\int \sqrt{x^2+9}\, dx$

75. $\displaystyle\int \sqrt{x}\,\cos\sqrt{x}\, dx$

76. $\displaystyle\int_{-1}^1 \frac{dx}{(2x+1)^3}$

77. $\displaystyle\int_e^{e^2} \ln x\sqrt{x\ln x-x}\, dx$

78. $\displaystyle\int_{-\infty}^{\infty} xe^{-|x|}\, dx$

79. $\displaystyle\int \csc^3 x\,\cot x\, dx$

80. $\displaystyle\int \sin^4(\frac{x}{4})\, dx$

81. $\displaystyle\int_0^{3/2} \frac{dx}{\sqrt{9-2x^2}}$

82. $\displaystyle\int \arcsin x\, dx$

83. $\displaystyle\int \sec^3\theta\, d\theta$

84. $\displaystyle\int \frac{x^3}{\sqrt{x^2-4}}\, dx$

85. $\displaystyle\int_0^{\infty} (x+1)e^{-x}\, dx$

86. $\displaystyle\int \tan^3 2x\,\sec^4 2x\, dx$

87. $\displaystyle\int_{1/\sqrt{2}}^{\sqrt{3}/2} \frac{\arccos x}{\sqrt{1-x^2}}\, dx$

88. $\displaystyle\int \frac{\tan x}{\ln\sec x}\, dx$

89. $\displaystyle\int e^x\sin 2x\, dx$

90. $\displaystyle\int_0^1 \frac{t^2}{(25-9t^2)^{3/2}}\, dt$

91. $\displaystyle\int \frac{\arctan x}{x^2}\, dx$

92. $\displaystyle\int \frac{\sin x\, e^{\sec x}}{\cos^2 x}\, dx$

93. $\displaystyle\int_1^e x^3\ln x\, dx$

94. $\displaystyle\int (\arcsin x)^2\, dx$

95. $\displaystyle\int_0^2 \frac{dx}{(x-1)^{2/3}}\, dx$

96. $\displaystyle\int \tan^3 2x\,\sec^3 2x\, dx$

97. $\displaystyle\int \frac{3x-2}{(x+1)^2(x-1)}\, dx$

98. $\displaystyle\int_{1/\sqrt{2}}^{\sqrt{2}} (1-t)\sqrt{2-t^2}\, dt$

99. $\displaystyle\int_1^{\infty} \frac{\arctan x}{x^2+1}\, dx$

100. $\displaystyle\int_1^{\sqrt{2}} x\,\text{arcsec}\, x\, dx$

101. $\displaystyle\int e^{x/2}\cos 3x\, dx$

102. $\displaystyle\int \frac{\sec^4 x}{\sqrt{\tan x}}\, dx$

103. $\displaystyle\int \frac{x^2}{x^4-1}\, dx$

104. $\displaystyle\int_0^1 \frac{dx}{(3x-1)^{2/3}}\, dx$

105. $\displaystyle\int_1^{\infty} \frac{dx}{x^2} - \int_0^1 \frac{dy}{y^{1/3}}$

106. $\displaystyle\int_0^{10} xe^{5x}\, dx$

Answers:

1. $\sqrt{x^2 - 1} + C$

2. $\frac{\cos ax}{a^2} + \frac{x \sin ax}{a} + C$

3. $2\ln|x+1| + 5\ln|x-2| + C$

4. $\frac{2}{3}$

5. $\frac{2}{7}(x+1)^{7/2} - \frac{6}{5}(x+1)^{5/2} + 2(x+1)^{3/2} - 2\sqrt{x+1} + C$

6. 1 (Improper)

7. $\frac{472}{15}$ (Improper)

8. $\frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + C$

9. $[1 - \ln|\cos\theta|]\cos\theta + C$

10. 2 (Improper)

11. 1 (Improper)

12. $\frac{1}{14}\sec^7 2\theta - \frac{1}{5}\sec^5 2\theta + \frac{1}{6}\sec^3 2\theta + C$

13. $\frac{142}{105}$

14. $\ln|x| + \frac{1}{x+1} - \ln|x+1| + C$

15. $\frac{\sqrt{3}}{4}$

16. Diverges. (Improper)

17. $\frac{1}{2}\ln|x| - \frac{1}{2}\ln|x+2| + C$

18. $(x+1)\arctan\sqrt{x} - \sqrt{x} + C$

19. $\frac{1}{e}$ (Improper)

20. $\frac{1}{3}(4-x^2)^{3/2} - 4\sqrt{4-x^2} + C$

21. Diverges. (Improper)

22. $\frac{186}{5}$

23. $\frac{1}{12}\sin^6 2\theta - \frac{1}{16}\sin^8 2\theta + C$

24. $\frac{2}{3}(8 - 5\sqrt{2})$

25. $\frac{1}{2}(x^2+1)\arctan x - \frac{1}{2}x + C$

26. Diverges. (Improper)

27. $\frac{1}{2}e^x(\sin x - \cos x) + C$

28. $\frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

29. $-\frac{1}{10}\cos^5 x^2 + \frac{1}{14}\cos^7 x^2 + C$

30. $-\frac{1}{10}\csc^5 2x + \frac{1}{6}\csc^3 2x + C$

31. Diverges. (Improper)

32. 1 (Improper)

33. $3\sin^3(\frac{x}{3}) - 2\sin^5(\frac{x}{3}) + C$

34. $\frac{\pi}{3}$

35. $[\ln|\cot x| + 1]\tan x + C$

36. $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

37. $\frac{1}{4}\frac{x}{\sqrt{9-4x^2}} - \frac{1}{8}\arcsin\frac{2x}{3} + C$

38. $\frac{\pi}{18}$ (Improper)

39. Diverges. (Improper)

40. $\frac{x}{8} - \frac{1}{32}\sin 4x + C$

41. $-\frac{1}{2}\cot^2 x + \ln|\csc x| + C$

42. $\frac{1}{4}\tan^4 x + \frac{1}{3}\tan^6 x + \frac{1}{8}\tan^8 x + C$

43. $1 - \frac{1}{\sqrt{2}}(1 + \ln\sqrt{2})$

44. $\sqrt{9x^2 - 4} - 2\operatorname{arcsec}\frac{3x}{2} + C$

45. $\frac{\pi}{12}$ (Improper)

46. -1 (Improper)

47. $\frac{1}{4}(\sec 2x \tan 2x - \ln|\sec 2x + \tan 2x|) + C$

48. $\frac{\sqrt{x^2-1}}{x} + C$

49. $\frac{x^3}{27}[9\ln^2 x - 6\ln x + 2] + C$

50. $\frac{2-\sqrt{2}}{3}$

51. 2 (Improper)

52. Diverges. (Improper)

53. $\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$

54. $\arctan e^x + C$

55. $-e^{-x}(x+1) + C$

56. $\frac{1}{4}[2\ln x + \sin(2\ln x)] + C$

57. $\frac{2}{13}e^{2x}\cos 3x + \frac{3}{13}e^{2x}\sin 3x + C$

58. $\frac{4}{3}(x^2-4)^{3/2} + \frac{1}{5}(x^2-4)^{5/2} + C$

59. $\frac{\pi}{2}$ (Improper)

60. Diverges. (Improper)

61. $\frac{1}{3}\ln|\tan 3x| + C$

62. $\frac{9}{16}\sqrt{4x^2 - 9} + \frac{1}{48}(4x^2 - 9)^{3/2} + C$

63. Diverges. (Improper)

64. 1 (Improper)

65. $-\frac{1}{x-1} + 2\ln|x-1| + 3\ln|x-2| + C$

66. $x\ln^2 x - 2x\ln x + 2x + C$

67. $\frac{1}{2}x^3(x^3+1)^{2/3} - \frac{3}{10}(x^3+1)^{5/3} + C$

68. $\frac{1}{8}\sin^4 2\theta + C$

69. Diverges. (Improper)

70. $\frac{\sqrt{5}}{8}$

71. $-\frac{1}{4}$ (Improper)

72. $x\tan x - \frac{1}{2}x^2 - \ln|\sec x| + C$

73. $x + \frac{1}{3}\tan^3 x - \tan x + C$

74. $\frac{1}{2}x\sqrt{x^2+9} + \frac{9}{2}\ln|\sqrt{x^2+9} + x| + C$

75. $2x\sin\sqrt{x} - 4\sin\sqrt{x} + 4\sqrt{x}\cos\sqrt{x} + C$

76. Diverges. (Improper)

77. $\frac{2e^3}{3}$

78. 0 (Improper)

79. $-\frac{1}{3}\csc^3 x + C$

80. $\frac{3x}{8} - \sin\frac{x}{2} + \frac{1}{8}\sin x + C$

81. $\frac{\pi}{4\sqrt{2}}$

82. $x\arcsin x + \sqrt{1-x^2} + C$

83. $\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$

84. $4\sqrt{x^2-4} + \frac{1}{3}(x^2-4)^{3/2} + C$

85. 2

86. $\frac{1}{8}\tan^4 2x + \frac{1}{12}\tan^6 2x + C$

87. $\frac{5\pi^2}{288}$

88. $\ln\ln\sec x + C$

89. $\frac{1}{5}e^x(\sin 2x - 2\cos 2x) + C$

90. $\frac{1}{27}\left[\frac{3}{4} - \arcsin\frac{3}{5}\right]$

91. $-\frac{1}{x}\arctan x + \ln|x| - \frac{1}{2}\ln(x^2 + 1) + C$

92. $e^{\sec x} + C$

93. $\frac{1}{16}(3e^4 + 1)$

94. $x(\arcsin x)^2 + 2\sqrt{1 - x^2}\,\arcsin x - 2x + C$

95. 6 (Improper)

96. $\frac{1}{10}\sec^5 2x - \frac{1}{6}\sec^3 2x + C$

97. $-\frac{5}{2(x+1)} - \frac{1}{4}\ln|x + 1| + \frac{1}{4}\ln|x - 1| + C$

98. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}(1 + \sqrt{2})$

99. $\frac{3\pi^2}{32}$ (Improper)

100. $\frac{1}{4}(\pi - 2)$

101. $\frac{2}{37}e^{x/2}(\cos 3x + 6\sin 3x) + C$

102. $2\sqrt{\tan x} + \frac{2}{5}(\tan x)^{5/2} + C$

103. $\frac{1}{4}\ln|x - 1| - \frac{1}{4}\ln|x + 1| + \frac{1}{2}\arctan x + C$

104. $1 + \sqrt[3]{2}$ (Improper)

105. $-\frac{1}{2}$ (Improper)

106. $\frac{1}{25}(49e^{50} + 1)$