

1 Curve Sketching

For the each problem determine the following if applicable:

- (a) x - and y -intercepts
- (b) vertical and horizontal asymptotes
- (c) open intervals on which the function is increasing and decreasing
- (d) open intervals where the function is concave up and concave down
- (e) coordinates of all relative extrema
- (f) coordinates of all points of inflection
- (g) sketch the curve, showing required information

1. $y = x^4 - 18x^2 + 56$

2. $y = \frac{2x - x^2}{x^2 - 2x + 1}$

3. $y = x^5 - 5x$

4. $y = \frac{(x - 3)(x + 2)}{(x + 1)(x - 2)}$

5. $y = x^4 - 4x^3$

6. $y = x^3 - 9x^2 + 15x - 5$

7. $y = x^3 - 3x^2 + 3$

8. $y = 2x^3 - 12x^2 + 18x$

9. $y = 2x^4 - x^2$

10. $y = \frac{1}{4}(x^3 - \frac{3}{2}x^2 - 6x + 2)$

11. $y = x^5 - 5x^4$

12. $y = 8 - 9x + 6x^2 - x^3$

13. $y = \frac{9x}{(3x + 1)^2}$
 $y' = \frac{9(1 - 3x)}{(3x + 1)^3}$ $y'' = \frac{54(3x - 2)}{(3x + 1)^4}$

14. $y = \frac{x^2}{x^2 - 1}$
 $y' = \frac{-2x}{(x^2 - 1)^2}$ $y'' = \frac{2(3x^2 + 1)}{(x^2 - 1)^3}$

15. $y = \frac{x}{(x + 1)^2}$
 $y' = \frac{1 - x}{(x + 1)^3}$ $y'' = \frac{2(x - 2)}{(x + 1)^4}$

16. $y = \frac{x^2 - 1}{x^3}$
 $y' = \frac{-x^2 + 3}{x^4}$ $y'' = \frac{2x^2 - 12}{x^5}$

17. $y = \frac{x + 1}{x^2}$
 $y' = \frac{-x - 2}{x^3}$ $y'' = \frac{2x + 6}{x^4}$

18. $f(x) = x^{2/3}(x - 5)$
 $f'(x) = \frac{5(x - 2)}{3x^{1/3}}$ $f''(x) = \frac{10(x + 1)}{9x^{4/3}}$

19. $y = \frac{1}{x^2 - x}$
 $y' = \frac{1 - 2x}{(x^2 - x)^2}$ $y'' = \frac{2(3x^2 - 3x + 1)}{(x^2 - x)^3}$

20. $y = x(16 - x^2)^{1/2}$
 $y' = 2(16 - x^2)^{-1/2}(8 - x^2)$
 $y'' = -2x(16 - x^2)^{-3/2}(24 - x^2)$

21. $f(x) = 18(x^{-2} + x^{-1}) = \frac{18(x + 1)}{x^2}$

22. $f(x) = x^{1/3}(x + 1)$
 $f'(x) = \frac{4x + 1}{2x^{2/3}}$ $f''(x) = \frac{2(2x - 1)}{9x^{5/3}}$

23. $f(x) = \frac{-6x}{x^2 + 1}$
 $f'(x) = \frac{6(x + 1)(x - 1)}{(x^2 + 1)^2}$
 $f''(x) = \frac{-12x(x^2 - 3)}{(x^2 + 1)^3}$

24. $y = 5x^{2/3} - x^{5/3}$

25. $f(x) = \frac{6 - 6x}{x^2}$
 $f'(x) = \frac{6x - 12}{x^3}$ $f''(x) = \frac{36 - 12x}{x^4}$

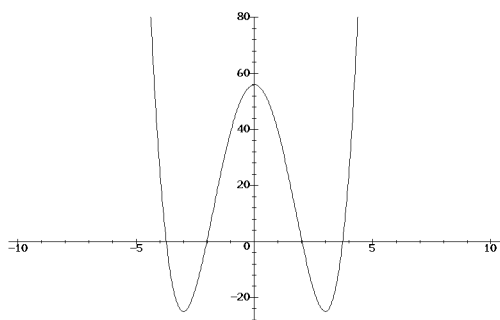
26. $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$
 $f'(x) = -\frac{1}{x^2} - \frac{2}{x^3}$ $f''(x) = \frac{2}{x^3} + \frac{6}{x^4}$

27. $y = \frac{x^2 - 1}{x^3}$
 $y' = \frac{3 - x^2}{x^4}$ $y'' = \frac{2(x^2 - 6)}{x^5}$

28. $y = x^4 - 6x^2 + 8x + 10$
 $\frac{dy}{dx} = 4(x - 1)^2(x + 2)$

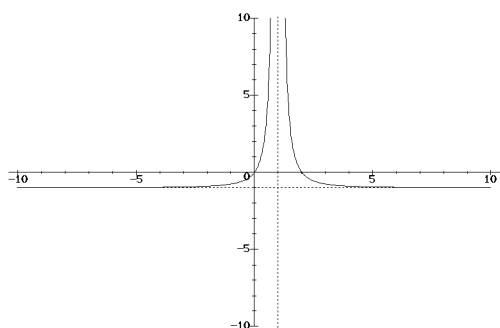
Answers:

1.



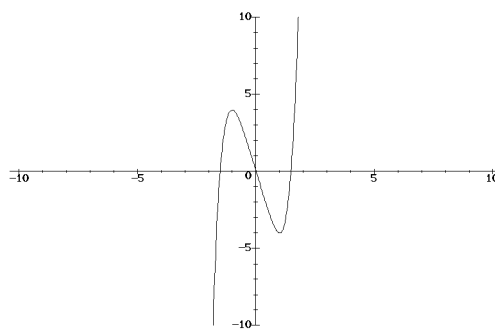
x -intercepts	$(\pm 2, 0); (\pm \sqrt{14}, 0)$
y -intercept	$(0, 56)$
H.A.	none
V.A.	none
Increasing on	$(-3, 0), (3, \infty)$
Decreasing on	$(-\infty, -3), (0, 3)$
Concave up on	$(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Concave down on	$(-\sqrt{3}, \sqrt{3})$
Rel. Max. at	$(0, 56)$
Rel. Min. at	$(-3, -25); (3, -25)$
Inflection pt. at	$(-\sqrt{3}, 11); (\sqrt{3}, 11)$

2.



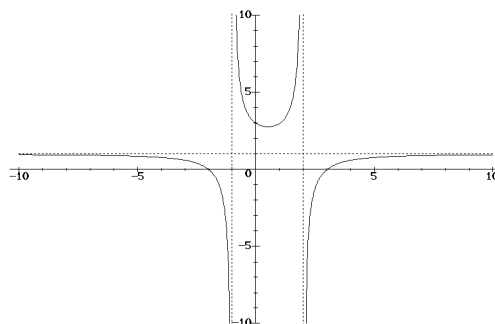
x -intercepts	$(0, 0); (2, 0)$
y -intercept	$(0, 0)$
H.A.	$y = -1$
V.A.	$x = 1$
Increasing on	$(-\infty, 1)$
Decreasing on	$(1, \infty)$
Concave up on	$(-\infty, 1), (1, \infty)$
Concave down on	none
Rel. Max. at	none
Rel. Min. at	none
Inflection pt. at	none

3.



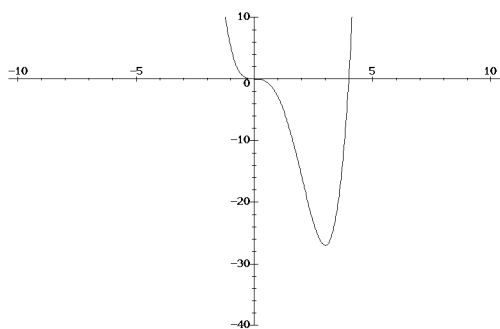
x -intercepts	$(0, 0); (-\sqrt{5}, 0); (\sqrt{5}, 0)$
y -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, -1), (1, \infty)$
Decreasing on	$(-1, 1)$
Concave up on	$(0, \infty)$
Concave down on	$(-\infty, 0)$
Rel. Max. at	$(-1, 4)$
Rel. Min. at	$(1, -4)$
Inflection pt. at	$(0, 0)$

4.



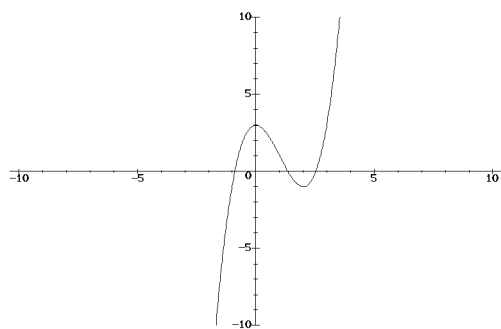
x -intercepts	$(-2, 0); (3, 0)$
y -intercept	$(0, 3)$
H.A.	$y = 1$
V.A.	$x = -1; x = 2$
Increasing on	$(\frac{1}{2}, 2), (2, \infty)$
Decreasing on	$(-\infty, -1), (-1, \frac{1}{2})$
Concave up on	$(-1, 2)$
Concave down on	$(-\infty, -1), (2, \infty)$
Rel. Max. at	none
Rel. Min. at	$(\frac{1}{2}, \frac{25}{9})$
Inflection pt. at	none

5.



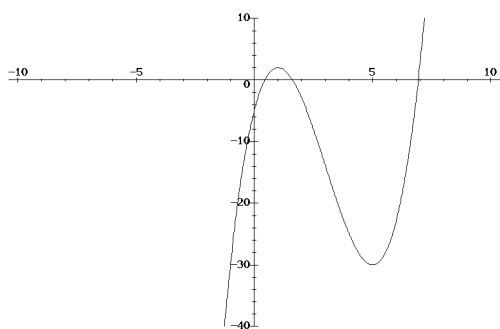
x -intercepts	$(0, 0); (4, 0)$
y -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(3, \infty)$
Decreasing on	$(-\infty, 3)$
Concave up on	$(-\infty, 0), (2, \infty)$
Concave down on	$(0, 2)$
Rel. Max. at	none
Rel. Min. at	$(3, -27)$
Inflection pt. at	$(0, 0); (2, -16)$

7.



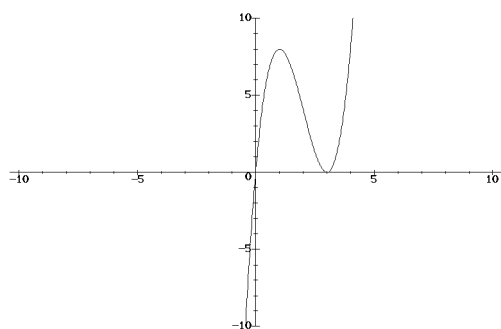
x -intercepts	irrational
y -intercept	$(0, 3)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, 0), (2, \infty)$
Decreasing on	$(0, 2)$
Concave up on	$(1, \infty)$
Concave down on	$(-\infty, 1)$
Rel. Max. at	$(0, 3)$
Rel. Min. at	$(2, -1)$
Inflection pt. at	$(1, 1)$

6.



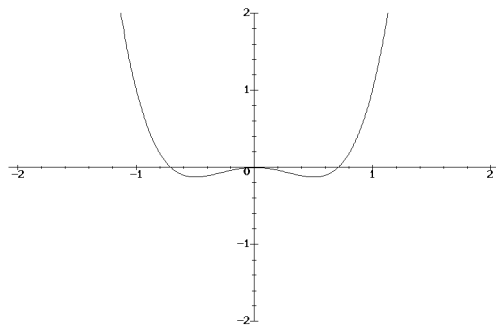
x -intercepts	irrational
y -intercept	$(0, -5)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, 1), (5, \infty)$
Decreasing on	$(1, 5)$
Concave up on	$(3, \infty)$
Concave down on	$(-\infty, 3)$
Rel. Max. at	$(1, 2)$
Rel. Min. at	$(5, -30)$
Inflection pt. at	$(3, -14)$

8.



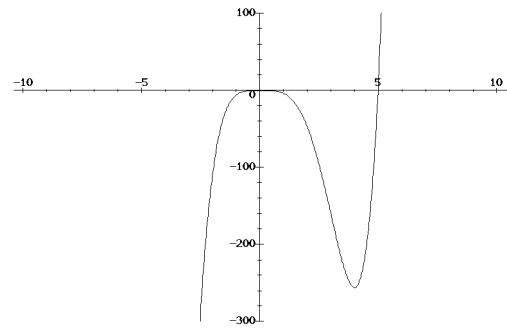
x -intercepts	$(0, 0); (3, 0)$
y -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, 1), (3, \infty)$
Decreasing on	$(1, 3)$
Concave up on	$(2, \infty)$
Concave down on	$(-\infty, 2)$
Rel. Max. at	$(1, 8)$
Rel. Min. at	$(3, 0)$
Inflection pt. at	$(2, 4)$

9.



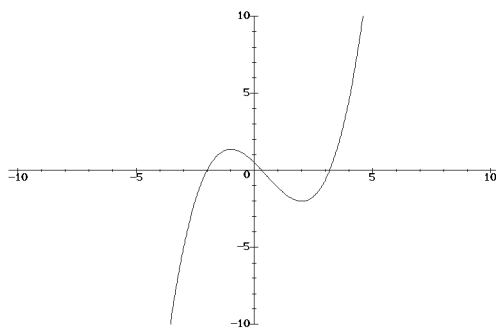
x -intercepts	$(0, 0); (-\frac{1}{\sqrt{2}}, 0); (\frac{1}{\sqrt{2}}, 0)$
y -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\frac{1}{2}, 0), (\frac{1}{2}, \infty)$
Decreasing on	$(-\infty, -\frac{1}{2}), (0, \frac{1}{2})$
Concave up on	$(-\infty, -\frac{1}{\sqrt{12}}), (\frac{1}{\sqrt{12}}, \infty)$
Concave down on	$(-\frac{1}{\sqrt{12}}, \frac{1}{\sqrt{12}})$
Rel. Max. at	$(0, 0)$
Rel. Min. at	$(-\frac{1}{2}, -\frac{1}{8}); (\frac{1}{2}, -\frac{1}{8})$
Inflection pt. at	$(-\frac{1}{\sqrt{12}}, -\frac{5}{72}); (\frac{1}{\sqrt{12}}, -\frac{5}{72})$

11.



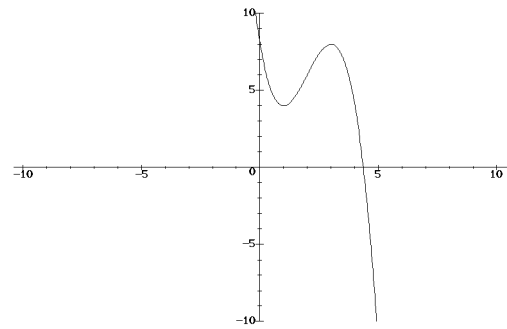
x -intercepts	$(0, 0); (5, 0)$
y -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, 0), (4, \infty)$
Decreasing on	$(0, 4)$
Concave up on	$(3, \infty)$
Concave down on	$(-\infty, 3)$
Rel. Max. at	$(0, 0)$
Rel. Min. at	$(4, -256)$
Inflection pt. at	$(3, -162)$

10.



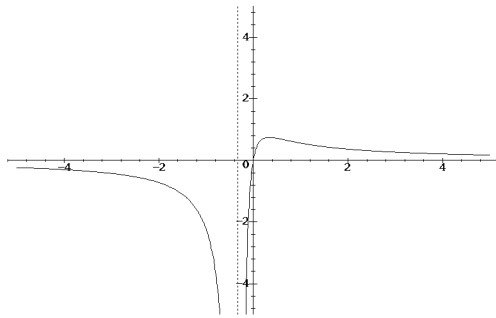
x -intercepts	$(-2, 0); (\frac{7 \pm \sqrt{33}}{4}, 0)$
y -intercept	$(0, \frac{1}{2})$
H.A.	none
V.A.	none
Increasing on	$(-\infty, -1), (2, \infty)$
Decreasing on	$(-1, 2)$
Concave up on	$(\frac{1}{2}, \infty)$
Concave down on	$(-\infty, \frac{1}{2})$
Rel. Max. at	$(-1, \frac{1}{8})$
Rel. Min. at	$(2, -2)$
Inflection pt. at	$(\frac{1}{2}, -\frac{5}{16})$

12.



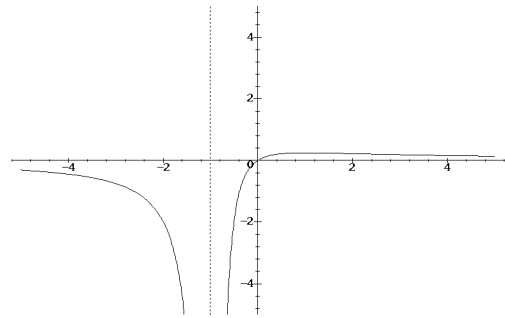
x -intercepts	irrational
y -intercept	$(0, 8)$
H.A.	none
V.A.	none
Increasing on	$(1, 3)$
Decreasing on	$(-\infty, 1), (3, \infty)$
Concave up on	$(-\infty, 2)$
Concave down on	$(2, \infty)$
Rel. Max. at	$(3, 8)$
Rel. Min. at	$(1, 4)$
Inflection pt. at	$(2, 6)$

13.



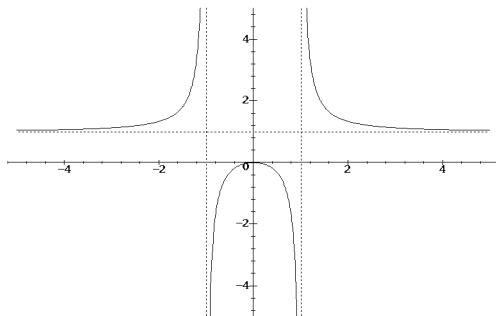
x -intercepts	$(0, 0)$
y -intercept	$(0, 0)$
H.A.	$y = 0$
V.A.	$x = -\frac{1}{3}$
Increasing on	$(-\frac{1}{3}, \frac{1}{3})$
Decreasing on	$(-\infty, -\frac{1}{3}), (\frac{1}{3}, \infty)$
Concave up on	$(\frac{2}{3}, \infty)$
Concave down on	$(-\infty, -\frac{1}{3}), (-\frac{1}{3}, \frac{2}{3})$
Rel. Max. at	$(\frac{1}{3}, \frac{3}{4})$
Rel. Min. at	none
Inflection pt. at	$(\frac{2}{3}, \frac{2}{3})$

15.



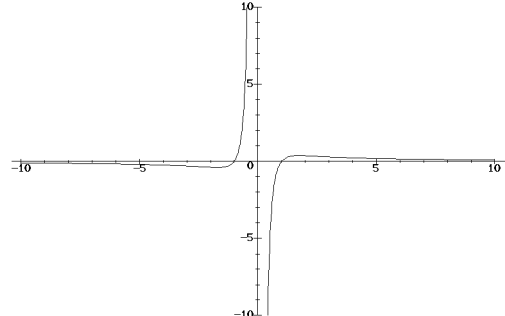
x -intercepts	$(0, 0)$
y -intercept	$(0, 0)$
H.A.	$y = 0$
V.A.	$x = -1$
Increasing on	$(-1, 1)$
Decreasing on	$(-\infty, -1), (1, \infty)$
Concave up on	$(2, \infty)$
Concave down on	$(-\infty, -1), (-1, 2)$
Rel. Max. at	$(1, \frac{1}{4})$
Rel. Min. at	none
Inflection pt. at	$(2, \frac{2}{9})$

14.



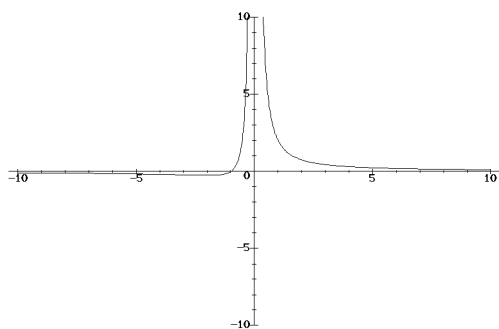
x -intercepts	$(0, 0)$
y -intercept	$(0, 0)$
H.A.	$y = 1$
V.A.	$x = \pm 1$
Increasing on	$(-\infty, -1), (-1, 0)$
Decreasing on	$(0, 1), (1, \infty)$
Concave up on	$(-\infty, -1), (1, \infty)$
Concave down on	$(-1, 1)$
Rel. Max. at	$(0, 0)$
Rel. Min. at	none
Inflection pt. at	none

16.



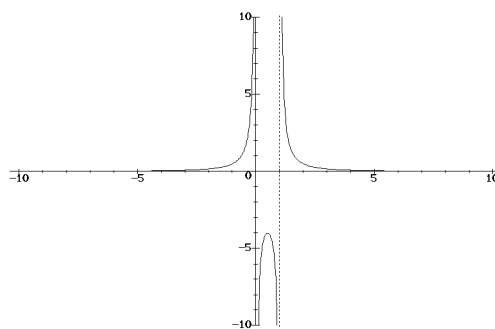
x -intercepts	$(\pm 1, 0)$
y -intercept	none
H.A.	$y = 0$
V.A.	$x = 0$
Increasing on	$(-\sqrt{3}, 0), (0, \sqrt{3})$
Decreasing on	$(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Concave up on	$(-\sqrt{6}, 0), (\sqrt{6}, \infty)$
Concave down on	$(-\infty, -\sqrt{6}), (0, \sqrt{6})$
Rel. Max. at	$(\sqrt{3}, \frac{2}{3\sqrt{3}})$
Rel. Min. at	$(-\sqrt{3}, -\frac{2}{3\sqrt{3}})$
Inflection pt. at	$(-\sqrt{6}, -\frac{5}{6\sqrt{6}}); (\sqrt{6}, \frac{5}{6\sqrt{6}})$

17.



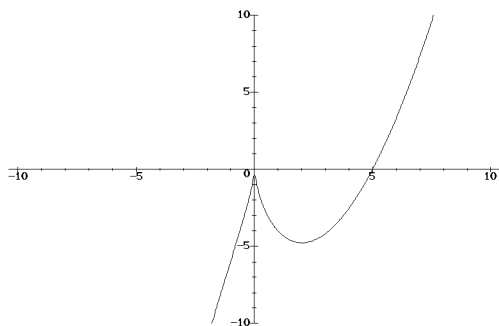
<i>x</i> -intercepts	$(-1, 0)$
<i>y</i> -intercept	none
H.A.	$y = 0$
V.A.	$x = 0$
Increasing on	$(-2, 0)$
Decreasing on	$(-\infty, -2), (0, \infty)$
Concave up on	$(-3, 0), (0, \infty)$
Concave down on	$(-\infty, -3)$
Rel. Max. at	none
Rel. Min. at	$(-2, -\frac{1}{4})$
Inflection pt. at	$(-3, -\frac{2}{9})$

19.



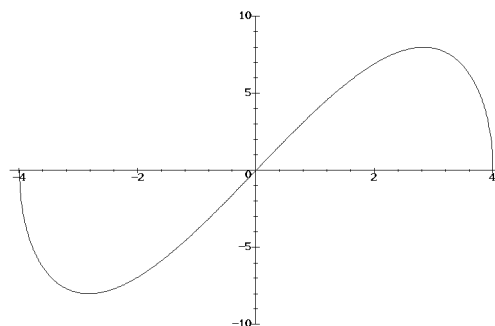
<i>x</i> -intercepts	none
<i>y</i> -intercept	none
H.A.	$y = 0$
V.A.	$x = 0; x = 1$
Increasing on	$(-\infty, 0), (0, \frac{1}{2})$
Decreasing on	$(\frac{1}{2}, 1), (1, \infty)$
Concave up on	$(-\infty, 0), (1, \infty)$
Concave down on	$(0, 1)$
Rel. Max. at	$(\frac{1}{2}, -4)$
Rel. Min. at	none
Inflection pt. at	none

18.



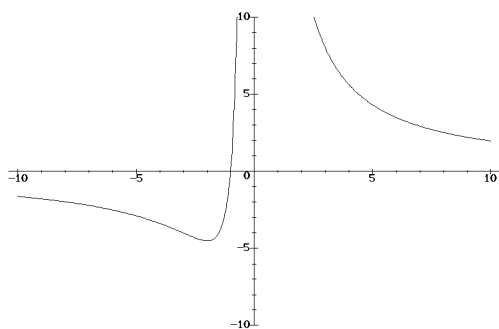
<i>x</i> -intercepts	$(0, 0); (5, 0)$
<i>y</i> -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\infty, 0), (2, \infty)$
Decreasing on	$(0, 2)$
Concave up on	$(-1, 0), (0, \infty)$
Concave down on	$(-\infty, -1)$
Rel. Max. at	$(0, 0)$
Rel. Min. at	$(2, -3\sqrt[3]{4})$
Inflection pt. at	$(-1, -6)$

20.



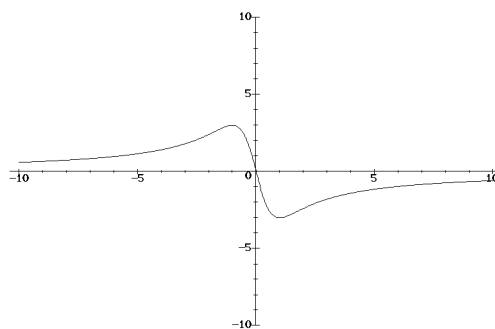
<i>x</i> -intercepts	$(0, 0); (\pm 4, 0)$
<i>y</i> -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\sqrt[3]{8}, \sqrt[3]{8})$
Decreasing on	$(-4, -\sqrt[3]{8}), (\sqrt[3]{8}, 4)$
Concave up on	$(-4, 0)$
Concave down on	$(0, 4)$
Rel. Max. at	$(\sqrt[3]{8}, 8)$
Rel. Min. at	$(-\sqrt[3]{8}, -8)$
Inflection pt. at	$(0, 0)$

21.



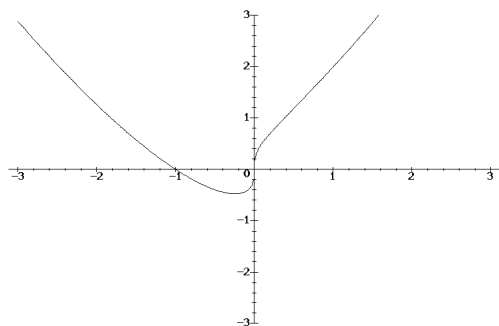
<i>x</i> -intercepts	$(-1, 0)$
<i>y</i> -intercept	none
H.A.	$y = 0$
V.A.	$x = 0$
Increasing on	$(-2, 0)$
Decreasing on	$(-\infty, -2), (0, \infty)$
Concave up on	$(-3, 0), (0, \infty)$
Concave down on	$(-\infty, -3)$
Rel. Max. at	none
Rel. Min. at	$(-2, -\frac{9}{2})$
Inflection pt. at	$(-3, -4)$

23.



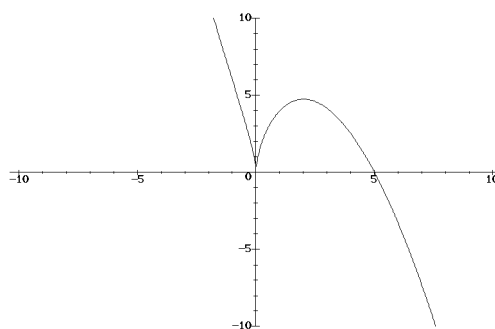
<i>x</i> -intercepts	$(0, 0)$
<i>y</i> -intercept	$(0, 0)$
H.A.	$y = 0$
V.A.	none
Increasing on	$(-\infty, -1), (1, \infty)$
Decreasing on	$(-1, 1)$
Concave up on	$(-\infty, -\sqrt{3}), (0, \sqrt{3})$
Concave down on	$(-\sqrt{3}, 0), (\sqrt{3}, \infty)$
Rel. Max. at	$(-1, 3)$
Rel. Min. at	$(1, -3)$
Inflection pt. at	$(0, 0); (-\sqrt{3}, \frac{3\sqrt{3}}{2}); (\sqrt{3}, -\frac{3\sqrt{3}}{2})$

22.



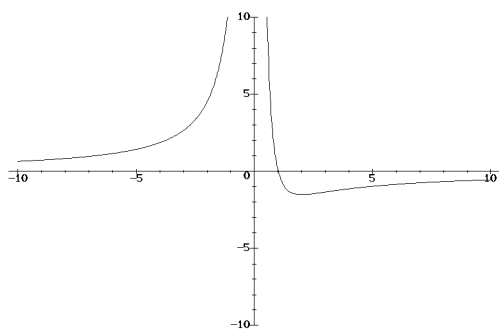
<i>x</i> -intercepts	$(0, 0); (-1, 0)$
<i>y</i> -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(-\frac{1}{4}, \infty)$
Decreasing on	$(-\infty, -\frac{1}{4})$
Concave up on	$(-\infty, 0), (\frac{1}{2}, \infty)$
Concave down on	$(0, \frac{1}{2})$
Rel. Max. at	none
Rel. Min. at	$(-\frac{1}{4}, -\frac{3}{\sqrt[3]{256}})$
Inflection pt. at	$(0, 0); (\frac{1}{2}, \frac{3}{\sqrt[3]{16}})$

24.



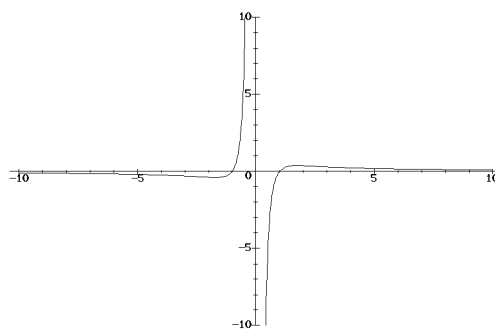
<i>x</i> -intercepts	$(0, 0); (5, 0)$
<i>y</i> -intercept	$(0, 0)$
H.A.	none
V.A.	none
Increasing on	$(0, 2)$
Decreasing on	$(-\infty, 0), (2, \infty)$
Concave up on	$(-\infty, -1)$
Concave down on	$(-1, 0), (0, \infty)$
Rel. Max. at	$(2, 3\sqrt[3]{4})$
Rel. Min. at	$(0, 0)$
Inflection pt. at	$(-1, 6)$

25.



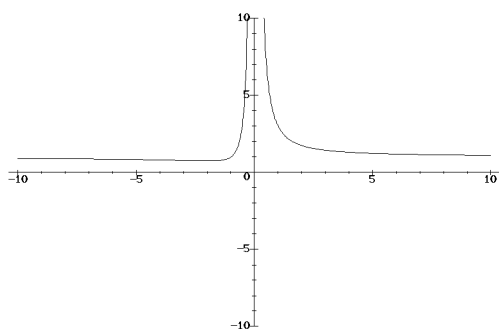
x -intercepts	$(1, 0)$
y -intercept	none
H.A.	$y = 0$
V.A.	$x = 0$
Increasing on	$(-\infty, 0), (2, \infty)$
Decreasing on	$(0, 2)$
Concave up on	$(-\infty, 0), (0, 3)$
Concave down on	$(3, \infty)$
Rel. Max. at	none
Rel. Min. at	$(2, -\frac{3}{2})$
Inflection pt. at	$(3, -\frac{4}{3})$

27.



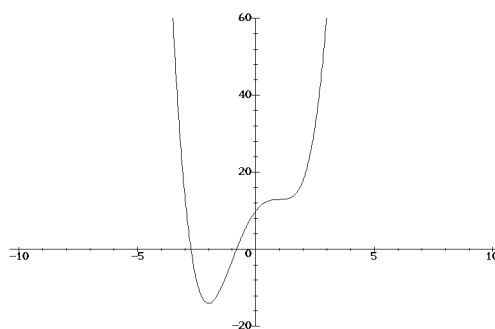
x -intercepts	$(\pm 1, 0)$
y -intercept	none
H.A.	$y = 0$
V.A.	$x = 0$
Increasing on	$(-\sqrt{3}, 0), (0, \sqrt{3})$
Decreasing on	$(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$
Concave up on	$(-\sqrt{6}, 0), ((\sqrt{6}, \infty)$
Concave down on	$(-\infty, -\sqrt{6}), (0, \sqrt{6})$
Rel. Max. at	$(\sqrt{3}, \frac{2}{3\sqrt{3}})$
Rel. Min. at	$(-\sqrt{3}, -\frac{2}{3\sqrt{3}})$
Inflection pt. at	$(-\sqrt{6}, -\frac{5}{6\sqrt{6}}); (\sqrt{6}, \frac{5}{6\sqrt{6}})$

26.



x -intercepts	none
y -intercept	none
H.A.	$y = 1$
V.A.	$x = 0$
Increasing on	$(-2, 0)$
Decreasing on	$(-\infty, -2), (0, \infty)$
Concave up on	$(-3, 0), (0, \infty)$
Concave down on	$(-\infty, -3)$
Rel. Max. at	none
Rel. Min. at	$(-2, \frac{3}{4})$
Inflection pt. at	$(-3, \frac{1}{9})$

28.



x -intercepts	irrational
y -intercept	$(0, 10)$
H.A.	none
V.A.	none
Increasing on	$(-2, \infty)$
Decreasing on	$(-\infty, -2)$
Concave up on	$(-\infty, -1), (1, \infty)$
Concave down on	$(-1, 1)$
Rel. Max. at	none
Rel. Min. at	$(-2, -14)$
Inflection pt. at	$(-1, -3); (1, 13)$