Sheet 1: domain, range, sets, limits and continuity

1. Given the following function

$$f(x,y) = 3x^2\sqrt{y} - 1,$$

compute the values $f(1,4), f(0,9), f(t^2,t)$ and $f(ab,9b)$. Which is the natural domain of f?

2. Let

$$f(x,y,z) = \sqrt{1-x^2-y^2-z^2},$$

compute the number $f(0,1/2,-1/2)$. Determine the natural domain of f.

3. Sketch the graphs of the following functions in the 3D-plane:

$$(a)\, f(x,y) = \sqrt{1-x^2-y^2} \qquad (b)\, f(x,y) = -\sqrt{x^2+y^2}.$$

4. Sketch the natural domain of the function

$$f(x,y) = \ln(x^2 - y).$$

5. Specify the boundary and the interior of the plane sets S whose points (x,y) satisfy the given conditions

$$(a)\ 0 < x^2+y^2 < 1, \qquad (b)\ x \geq 0, y < 0,$$
$$(c)\ x + y = 1, \qquad (d)\ |x| + |y| \leq 1.$$

Is S open, closed or neither?

6. Evaluate the following limits or, if appropriate, explain why they do not exist:

$$(a) \lim_{(x,y)\to(1,3)} xy + x^2, \qquad (b) \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{y}$$
$$(c) \lim_{(x,y)\to(0,0)} \frac{y^3}{x^2+y^2}, \qquad (d) \lim_{(x,y)\to(0,0)} \frac{\sin(x-y)}{\cos(x+y)},$$
$$(e) \lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2}, \qquad (f) \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2+y^4},$$
$$(g) \lim_{(x,y)\to(1,2)} \frac{2x^2-xy}{4x^2-y^2}, \qquad (h) \lim_{(x,y)\to(0,0)} \frac{xy^2}{(x^2+y^2)^{3/2}}.$$

7. Determine whether or not the functions of examples 1,2,3 and 4 considered in the lecture (section 2.2 in the notes) are continuous at the points where their limits were computed. In the cases in which the functions are not continuous, is it possible to re-define the functions in such a way that they become continuous? How?

8. What conditions must the nonnegative integers m, n and p satisfy to guarantee that the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^m y^n}{(x^2+y^2)^p},$$

exists. For these values, how can we define $f(0,0)$ so that the function is continuous in the whole xy-plane?