Cotorsion theories and $\Sigma$-pure injective cotilting modules

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Let $R$ be a ring. Given a cotorsion pair $\mathcal{A} := (\mathcal{A}, \mathcal{A}^\perp)$ in the category of right $R$-modules $\text{Mod-}R$, we denote by $\mathcal{A}_\ell$ the cotorsion pair generated by the modules $M$ in $\mathcal{A}$ which admit a resolution

$$P_\ell \to \ldots \to P_1 \to P_0 \to M \to 0$$

with $P_i$, $0 \leq i \leq \ell$ finitely generated projective modules. We compare the cotorsion theories $\mathcal{A}_\ell$, looking for conditions which guarantee the equalities $\mathcal{A}_\ell = \mathcal{A}_{\ell+n}$, $n \in \mathbb{N}$, and / or $\mathcal{A}_\ell = \mathcal{A}$. Particular emphasis is dedicated to cotorsion theories cogenerated by a cotilting module in the Noetherian case.