Leech lattice, Conway group $\text{Co}_2$ and associated binary codes

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The Leech lattice is a certain 24-dimensional $\mathbb{Z}$-submodule of the Euclidean space $\mathbb{R}^{24}$ whose automorphism group is the double cover $2\text{Co}_1$ of the Conway group $\text{Co}_1$. The Conway groups $\text{Co}_2$ and $\text{Co}_3$ are stabilizers of sublattices of the Leech lattice. We give a brief discussion of the Conway group $\text{Co}_2$. The group $\text{Co}_2$ admits a 23-dimensional indecomposable representation over $\mathbb{GF}(2)$ obtained from the 24-dimensional Leech lattice by reducing modulo 2 and factoring out a fixed vector. On the other hand, reduction modulo 2 of the 23-dimensional ordinary irreducible representation results in a decomposable 23-dimensional $\mathbb{GF}(2)$-representation. We construct this decomposable 23-dimensional $\mathbb{GF}(2)$-representation as a binary code. Furthermore, we show that this code contains a binary code of dimension 22 invariant and irreducible under the action of $\text{Co}_2$. 