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Root systems, toric arrangements and representation theory

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Kostant's partition function counts in how many ways a vector can be written as a sum of positive roots. It plays an important role in representation theory, since it yields effective computation of weight multiplicities and Littlewood-Richardson coefficients. Its values can be computed in a geometric way, by associating to the root system a "toric arrangement", as De Concini and Procesi have shown. The complement of this arrangement is known as the set of "regular points" of the torus, and its cohomology is a direct sum of contributions given by the "components" of the arrangement. The Weyl group acts naturally on these components; I will show how this action can be described by the combinatorics of affine Dynkin diagrams. In this way the components of the arrangement are counted, and then the Poincaré polynomial can be explicitly computed; in particular the absolute value of the Euler characteristic is shown to be equal to the order of the Weyl group. Moreover this yields a W-equivariant decomposition of the cohomology. Finally I will sketch possible applications of my work to some open problems.