On Lie solvable group algebras

Tibor Juhász

Eszterházy Károly College, Eger
{juhaszt@eke.hu}

The group algebra $FG$ of a group $G$ over a field $F$ may be considered as a Lie algebra with the Lie product $[x, y] = xy - yx$, where $x, y \in FG$. Denote by $[X, Y]$ the additive subgroup generated by all Lie products $[x, y]$ with $x \in X$ and $y \in Y$. We say that $FG$ is strongly Lie solvable if the strong Lie derived series $\delta^{(n)}(FG) = \left[\delta^{(n-1)}(FG), \delta^{(n-1)}(FG)\right]FG$ with $\delta^{(0)}(FG) = FG$ vanishes for some integer $n$; the minimal such integer is called the strong Lie derived length of $FG$ and is denoted by $dl^L(FG)$. Furthermore, $FG$ is called Lie solvable, if some of the terms of the Lie derived series $\delta^{(m)}(FG) = \left[\delta^{(n-1)}(FG), \delta^{(n-1)}(FG)\right]$ with $\delta^{(0)}(FG) = FG$ contain only the zero element. Denote by $dl_L(FG)$ the minimal element of the set $\{m \in \mathbb{N} \mid \delta^{(m)}(FG) = 0\}$, which is said to be the Lie derived length of $FG$.

Although there exist criteria for both Lie and strongly Lie solvability of group algebras, the exact values of $dl^L(FG)$ and $dl_L(FG)$ are known only in a few cases. The characterization of groups satisfying $dl_L(FG) = dl^L(FG)$ is also an open problem. This talk will be devoted to summarize our related results on this topic. Furthermore, we will have a try to extend the concept of Lie solvability for subsets of group algebras, and determine the Lie derived length of some well-known subsets.