Injective modules over principal left and right ideal domains

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Over a noetherian ring every injective module is a direct sum of indecomposable injective modules classified as either “tame” or “wild”. The tame ones are uniquely determined by the prime ideals of the ring and are now relatively well understood. Much less is known about wild injectives.

In this talk, we will consider (wild) indecomposable injective left modules over a principal left and right ideal domain. The internal “layered” structure of such an object can be described in two ways: first as the union of its socle series, and secondly, as the union of its elementary socle series, a concept from model theory introduced by Herzog in 1993 as the elementary analogue of the socle series of a module. At the same time, an indecomposable injective left module as a right module over its endomorphism ring is also the union of its socle series. Furthermore, we will see that there is a rich relationship between these descriptions of the module.