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The theories of relative torsion

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The theories of torsion are studied in the abelian categories and especially in the modules categories. The theories of relative torsion can be defined in arbitrary categories, in particulary in the category $\mathcal{C}_2\mathcal{V}$ of Hausdorff vectorial topological local convex spaces or in category of uniforms spaces. Let be a full subcategory Z of category $\mathcal{C}$. The morphism $f : X \longrightarrow Y \in \mathcal{C}$ is named Z - morphism if it exists an object $A \in\mid Z \mid$ and the morphisms $g : X \longrightarrow A$, $h : A \longrightarrow Y$ so that $f = hg$.

The monomorphism $m : Q \longrightarrow X$ is named Z - ker of morphism $f \in \mathcal{C}$, if fm is an Z - morphism and every morphism g for which fg is an Z - morphism has the form $g = mh$ for any morphism h.

It is noted $m = Z$ - *ker* f. The dual notion: $q = z$ - *coker* f. The row

$$X \stackrel{m}{\to} Y \stackrel{e}{\to} Z$$

is named Z - exact if $m = Z$ - *ker* e and $e = Z$ - *coker* m.

Definition[B1]. *The pair of subcategories $(\mathcal{K}, \mathcal{R})$ of category $\mathcal{C}$ is named the theory of relative torsion if:*

1. *$\mathcal{R}$ is a coreflective subcategory of the category $\mathcal{C}$ with the coreflective functor $k : \mathcal{C} \longrightarrow \mathcal{K}$.*
2. *$\mathcal{R}$ is a reflective subcategory of the category $\mathcal{C}$ with the reflective functor $r : \mathcal{C} \longrightarrow \mathcal{R}$.*
3. *For every object X of category $\mathcal{C}$ the row*

$$kX \stackrel{k^X}{\to} X \stackrel{r^X}{\to} rX$$

is $(\mathcal{K} \cap \mathcal{R})$ - exact, where k^X is $\mathcal{K}$ - coreplique and r^X is $\mathcal{R}$ - replique of object X.

In the article [B1] are shown the most important properties of the theories of relative torsion, in [B2]- are studied these theories for the category of uniforms spaces, in [B3-B5] for the category $\mathcal{C}_2\mathcal{V}$ with the condition that $\Gamma_0 \subset \mathcal{R}$ where Γ_0 is a subcategory of the complete spaces.

We note $\mathbb{K}_u$ the class of all coreflective subcategories $\mathcal{K}$ of the category $\mathcal{C}_2\mathcal{V}$ that contain the subcategory $\mathcal{M}$ of the space with Mackey topology: $\mathcal{M} \subset \mathcal{K}$; and $\mathbb{R}_b$ is the class of all reflective subcategories $\mathcal{R}$ which contain the subcategory $\mathcal{S}$ of the spaces with weak topology: $\mathcal{S} \subset \mathcal{R}$. We will examine the properties of the pairs $(\mathcal{K}, \mathcal{R})$ with $\mathcal{K} \in \mathbb{K}_u$ and $\mathcal{R} \in \mathbb{R}_b$.

Let be $\mathcal{K} \in \mathbb{K}_u$ and $\mathcal{R} \in \mathbb{R}_b$. For $X \in\mid \mathcal{C}_2\mathcal{V} \mid$ let be $r^X : X \longrightarrow rX$, $k^X : kX \longrightarrow X$ and $k^{rX} : krX \longrightarrow rX$ $\mathcal{R}$ - replique and $\mathcal{K}$ the coreplique of these respective objects. Then

$$r^X k^X = k^{rX} k(r^X)$$

Theorem. *The pair* $(\mathcal{K},\mathcal{R})$ *is a theory of relative torsion if:*

1. $r(\mathcal{K})\subset\mathcal{K}$*;*

2. $kr\sim rk$*;*

3. $r^X k^X = k^{rX}k(r^X)$ *is a pullback and poushot.*

Theorem. *1. For any element* $\mathcal{R}\in\mathbb{R}_b$ *exists an element* $\mathcal{K}=\varphi(\mathcal{R})\in\mathbb{K}_u$ *so that:*

a) $r(\mathcal{K})\subset\mathcal{K}$*;*

b) $kr\sim rk$*;*

c) $r^X k^X = k^{rX}k(r^X)$ *is a pullback.*

2. Let be $\mathcal{K}_1\in\mathbb{K}_u$*. If the pair* $(\mathcal{K}_1,\mathcal{R})$ *verifies the conditions a)- c) then* $\varphi(\mathcal{R})\subset\mathbb{K}_1$*.*

3. For any element $\mathcal{K}\in\mathbb{K}_u$ *exists an element* $\mathcal{R}=\psi(\mathcal{K})\in\mathbb{R}_b$ *so that the pair* $(\mathcal{K},\psi(\mathcal{K}))$ *has the dual properties.*

Theorem. *1.* $\psi\varphi\psi=\psi$*,* $\varphi\psi\varphi=\varphi$*.*

2. $\mathcal{R}\subset\psi\phi(\mathcal{R})$*,* $\mathcal{K}\subset\varphi\psi(\mathcal{K})$*.*

3. Let $\mathcal{R}_1\subset\mathcal{R}_2$*. Then* $\varphi(\mathcal{R}_2)\subset\varphi(\mathcal{R}_1)$

4. Let $\mathcal{K}_1\subset\mathcal{K}_2$*. Then* $\varphi(\mathcal{K}_2)\subset\varphi(\mathcal{K}_1)$

5. For any $\mathcal{R}\in\mathbb{R}_b$ *and* $\mathcal{K}\in\mathbb{K}_u$ *the pairs* $(\varphi(\mathcal{R}),\psi\varphi(\mathcal{R}))$ *and* $(\varphi\psi(\mathcal{K}),\psi(\mathcal{K}))$ *are theories of the relative torsion.*

References

[B1] Botnaru D., The theories of relative torsion in the categories, Mathematic Researches, 1984, Nr 76, p. 3-13 (in Russian).

[B2] Botnaru D., The theories of relative torsion in the categories of separated uniform spaces, Mathematic Researches, 1985, Nr 85, p. 43-57 (in Russian).

[B3] Botnaru D., The completion lattices of the categories of separated local convex spaces, Baku, International Topological Conference proceedings, 1985, p. 51-59 (in Russian).

[B4] Botnaru D., The theories of relative torsion in the categories of separated local convex spaces, Mathematic Researches, 1986, Nr 90, p. 28-40 (in Russian).

[B5] Botnaru D., The theories of relative torsion in the categories of separated local convex spaces, Researches of Differential Equations and Mathematical Analysis, Kishinev, 1988, p. 18-25 (in Russian).