The theories of relative torsion

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The theories of torsion are studied in the abelian categories and especially in the modules categories. The theories of relative torsion can be defined in arbitrary categories, in particular in the category $C_2V$ of Hausdorff vectorial topological local convex spaces or in category of uniforms spaces. Let be a full subcategory $Z$ of category $C$. The morphism $f : X \to Y \in C$ is named $Z$-morphism if it exists an object $A \in |Z|$ and the morphisms $g : X \to A$, $h : A \to Y$ so that $f = hg$.

The monomorphism $m : Q \to X$ is named $Z$-ker of morphism $f \in C$, if $fm$ is a $Z$-morphism and every morphism $g$ for which $fg$ is an $Z$-morphism has the form $g = mh$ for any morphism $h$.

It is noted $m = Z$-ker $f$. The dual notion: $q = z$-coker $f$. The row

$$X \xrightarrow{m} Y \xrightarrow{e} Z$$

is named $Z$-exact if $m = Z$-ker $e$ and $e = Z$-coker $m$.

Definition[B1]. The pair of subcategories $(K, R)$ of category $C$ is named the theory of relative torsion if:

1. $R$ is a coreflective subcategory of the category $C$ with the coreflective functor $k : C \to K$.
2. $R$ is a reflective subcategory of the category $C$ with the reflective functor $r : C \to R$.
3. For every object $X$ of category $C$ the row

$$kX \xrightarrow{k^X} X \xrightarrow{r^X} rX$$

is $(K \cap R)$ - exact, where $k^X$ is $K$-coreplique and $r^X$ is $R$-replique of object $X$.

In the article [B1] are shown the most important properties of the theories of relative torsion, in [B2]-are studied these theories for the category of uniforms spaces, in [B3-B5] for the category $C_2V$ with the condition that $\Gamma_0 \subset R$ where $\Gamma_0$ is a subcategory of the complete spaces.

We note $K_u$ the class of all coreflective subcategories $K$ of the category $C_2V$ that contain the subcategory $M$ of the space with Mackey topology: $M \subset K$; and $R_b$ is the class of all reflective subcategories $R$ which contain the subcategory $S$ of the spaces with weak topology: $S \subset R$. We will examine the properties of the pairs $(K, R)$ with $K \in K_u$ and $R \in R_b$.

Let be $K \in K_u$ and $R \in R_b$. For $X \in |C_2V|$ let be $r^X : X \to rX$, $k^X : kX \to X$ and $k^rX : krX \to rX$ - $R$-replique and $K$ the coreplique of these respective objects. Then

$$r^Xk^X = k^rXk(r^X)$$
**Theorem.** The pair \((K, R)\) is a theory of relative torsion if:

1. \(r(K) \subset K\);
2. \(kr \sim rk\);
3. \(r^X \overset{k}{=} k^r X k(rX)\) is a pullback and poushot.

**Theorem.**

1. For any element \(R \in R_b\) exists an element \(K = \varphi(R) \in K_u\) so that:
   a) \(r(K) \subset K\);
   b) \(kr \sim rk\);
   c) \(r^X \overset{k}{=} k^r X k(rX)\) is a pullback.

2. Let be \(K_1 \in K_u\). If the pair \((K_1, R)\) verifies the conditions a)-c) then \(\varphi(R) \subset K_1\).

3. For any element \(K \in K_u\) exists an element \(R = \psi(K) \in R_b\) so that the pair \((K, \psi(K))\) has the dual properties.

**Theorem.**

1. \(\psi \varphi \psi = \psi\), \(\varphi \psi \varphi = \varphi\).
2. \(R \subset \psi \phi(R), K \subset \varphi \psi(K)\).
3. Let \(R_1 \subset R_2\). Then \(\varphi(R_2) \subset \varphi(R_1)\)
4. Let \(K_1 \subset K_2\). Then \(\varphi(K_2) \subset \varphi(K_1)\)
5. For any \(R \in R_b\) and \(K \in K_u\) the pairs \((\varphi(R), \psi \varphi(R))\) and \((\varphi \psi(K), \psi(K))\) are theories of the relative torsion.

**References**


