INFINITE-DIMENSIONAL VERSIONS OF CLASSICAL REDUCTIVE LIE GROUPS

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We are going to discuss some topological groups which are associated with certain operator algebras on Hilbert spaces in just the same way as the reductive linear groups $\mathrm{GL}(n,\mathbb{C})$, $\mathrm{O}(n,\mathbb{C})$, $\mathrm{Sp}(n,\mathbb{C})$, their classical real forms and their covering groups are associated with the matrix algebra $M_n(\mathbb{C})$. Two kinds of problems concerning these groups naturally arise and will be addressed:

(1) the ones suggested by the classical theory, such as the existence of Cartan decompositions, Iwasawa decompositions, and appropriate versions of invariant measures;
(2) the ones that have no classical correspondent, for instance the existence of weaker topologies which still make these groups into topological groups with Lie algebras.

The framework for our discussion is the theory of noncommutative integration with respect to semifinite traces on von Neumann algebras. The classical instances of this framework are given by the matrix trace on $M_n(\mathbb{C})$ whose values on the projections are $0, 1, 2, \ldots, n$, and this is just the setting of the classical matrix groups; and the operator trace on $\mathcal{B}(\mathcal{H})$, whose values on projections are $0, 1, 2, \ldots, \infty$, and this leads to the classical Banach-Lie groups of operators on the complex Hilbert space $\mathcal{H}$. Many other traces on operator algebras are known, and they correspond to continuous geometries, in the sense that their range of values on projections is $[0, 1]$ or $[0, \infty]$. All these operator algebras give rise —in the way suggested above— to wide classes of examples from which a reasonably general theory of infinite-dimensional reductive Lie groups could eventually emerge.

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