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# Henri Poincaré. A life at the service of science

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Abstract. We give an overview about the life, personality and main scientific and philosophical contributions of Henri Poincaré.

# 1 Introduction

In 1954, the scientific community celebrated the 100th anniversary of Henri Poincaré's birth. At this time, Poincaré's fame was not at his highest point among mathematicians, and the spirit of Hilbert was dominating most mathematical minds. It was not at his highest point among physicists, with a physics essentially concerned with quantum theory.

Despite of this, the celebration was important in the many places where Poincaré's presence or name had been significant, and their content was
published as a Golden Book, reproduced in the last volume of Poincaré's Scientific Work.

We celebrate this year the 150th anniversary of Poincaré's birth, and Poincaré's popularity has reached new summits in the scientific world, and even for the layman. Chaos theory and the origins of special relativity have brought Poincaré's name and portraits in most popular science journals. However the celebration looks more confidential, making this conference most welcome.

Several new books about Einstein have been added this year to an already long list, but we are still waiting for a detailed biography of Poincaré. Therefore, an impressionist introduction to Henri Poincaré, the man and the scientist, may be useful, before the precise description of his achievements and his legacy given in the following lectures.

# 2 Family, childhood, studies

Poincaré is born on April 29, 1854 in Nancy, in the Hôtel Martigny, a town mansion
transformed in drugstore, which still exists at the corner of Grande-Rue and rue de Guise. Poincaré's family is well-known in Lorraine. His grand father, on the father side, Jacques-Nicolas is pharmacist, his father, Léon, a neurologist, is professor at the Faculty of Medicine, his uncle Antoni (the father of Raymond, future president of French Republic), a polytechnician, is general inspector of roads and bridges. Henri's mother, born Eugénie Launois, comes from a family of gentlemen farmers in Arrancy. Henri Poincaré's younger sister, Aline, will marry the famous philosopher Emile Boutroux, and their son Pierre will be a talented mathematician and philosopher.

With the exception of a dangerous diphteria when he is five years old, Poincaré's chilhood recalls the ones described in old-fashioned edifying books. A clever private tutor and the games invented for his sister and cousins reveal Henri's unlimited imagination and feed his fantastic memory. At the high school in Nancy (the future *Lycée Henri-Poincaré*), he is soon noticed as a first class student, qualified of 'mathematical monster' in the last years. Batchelor in letters and in science, he is already famous during the two years in mathematics preparing to the entrance concours to the 'Grandes Ecoles'.

Admitted at the fifth rank at the *Ecole Normale Supérieure* and at the first one at the *Ecole Polytechnique*, the young Henri chooses the last one, leaving

it at the second rank. He then goes to the Mining School (*Ecole des Mines*), where cristallography only meets his mathematical taste, and may have inspired his constant interest for group theory. Without the permission to follow the lectures, Poincaré gets his diploma in mathematics from the Faculty of Science of Paris in August 1876.

During the last two years in the Mining School, Poincaré prepares his PhD thesis in mathematics, defended on August 1rst, 1879 at the Faculty of Science, in front of a jury made of Bonnet, Bouquet et Darboux. It extends to partial differential equations some classical results of Briot and Bouquet on singular ordinary differential equations. Darboux's report, very positive about the results and the methods, is far less enthusiastic about the clarity of the style.

# 3 Carreer and personality

Poincaré starts working as a mining engineer in Vésoul in April 1879. After a carreer of several months, leading him to pay a dangerous visit to the Magny pit, where a firedamp explosion has killed sixteen workers, Poincaré will remain all his life on leave (but promoted !) in the *Corps des Mines*.

His academic life starts at the Faculty of Science of Caen, where he teaches analysis from 1879. Two years later, he moves to Paris, as lecturer of analysis at the Faculty of Science. He is successively appointed lecturer in physical mechanics and experimental physics in 1885, professor of mathematical physics and probability in 1886, professor of mathematical astronomy and celestial mechanics in 1896. He also teaches astronomy at the *Ecole Polytechnique* and theoretical electricity at the *Ecole des Postes et Télégraphes*. He has been a member of the
*Bureau des Longitudes*.

His former students describe Poincaré as a more devoted than brilliant teacher. According to Robert d'Adhémar :

> ¿From the beginning, the blackboard was covered with formulas and one had an extraordinary feeling of power; the words came fast and without hesitation. The lectures were infinitely austere.

According to Maurice d'Ocagne :

> One could not say that Poincaré was a marvellous professor. He did not have the orator's gifts required by excellence in teaching.

According to Léon Brillouin :

> I have seen him many times leaving his notes, anouncing he would try another method and improvising in front of us at the blackboard.

According to Louis Bourgoin :

> Poincaré was, in 1910 and 1911, a fashionable scientist attracting the mundane Paris crowd to listen him. During the first lectures, the room was more than full, but rapidly and happily, the crowd decreased soon. From the third lecture, only a few students and a few passionates remained. Poincaré always ended with simple formulas, translated in a language full of imagery, that we were forced to understand.

A detailed analysis by Dr. Toulouse gives interesting information about the man Henri Poincaré when 43 years old :

> Poincaré is 1 m. 65 high, weights 70 kilos, bent, a little big-bellied, with colored face, big nose and red, chestnut haired and blond moustache.
>
> He does not smoke and never tried.
>
> He does not seem to be chilly or specially sensitive to temperature, but he is subject to frequent colds. He does not sleep with an open window.
>
> He looks constantly absent minded. When speaking to him, one has the feeling that he has not followed or understood what was said, although he answers or thinks about the question.
>
> He believes to have a quiet, sweet and equal character. But he has no patience in his actions, and even in his work.
>
> He is neither passionate for his feelings, nor for his ideas, and he is neither sociable nor ready for confidences.
>
> In practical life, he is disciplined. He is not ordered, but appreciates the value of this quality.
>
> He speaks correctly, but with some shyness he is aware of. Hence he avoids to speak publiclly without preparation, except in scientific meetings. Before giving a speach, he prepares a number of sentences, but does not learn in by heart.
>
> He does not play chess and believes he could not be a good player. He does not hunt.

The popular magazine *L'Illustration* confirms this picture in 1912 :

> Very simple, very affable, looking somewhat in the clouds, he gave the picture of some type of quite unusual average between the mathematician of the new school, looking if not like an artist, at least very parisian, and the classical mathematician, forbidding and absorbed in his equations.

# 4 The *annus mirabilis*

The stay in Caen is certainly Poincaré's double *annus mirabilis*. Between August 1879 and October 1881, Poincaré not only marries Louise Poullain d'Andecy

(April 20 1881) (they will have three daughters, Jeanne, Yvonne, Henriette, followed by one son Léon), but sends more than twenty notes to the *Comptes Rendus de l'Académie des Sciences de Paris*, dealing with three completely different topics : the arithmetics of forms, the qualitative theory of differential equations, and automorphic functions.

The study of quadratic and ternary forms is inspired by the work of Charles Hermite, who reigns on French mathematics in this time, has teached analysis to Poincaré at the Ecole Polytechnique, and owes his reputation, among other things, to his proof of the transcendental character of the number $e$. Hermite reacts very enthusiastically to Poincaré's work, even if Poincaré's introduction of non-Euclidian geometry in the study of ternary forms completely disgusts the old analyst, who always hated geometry. Hermite suggests to Poincaré to read Kronecker's work ('without omitting nothing'), and makes ignored suggestions for improving Poincaré's style.

Poincaré has told himself how he discovered the automorphic functions, and I will not recall the famous story of the omnibus. The automorphic functions, which extend the (periodic) trigonometric functions and the (doubly periodic) elliptic functions, recover their values under the action of a discrete group of homographic substitutions. The corresponding

tessellation of the complex plane, made by rectangles for elliptic functions, is replaced by curvilinear figures bounded by curves that Poincaré identifies with the 'straight lines' in a new model of Lobatchevsky's geometry. Striking illustrations can be found in some

paintings of Escher. In Göttingen, Félix Klein, trying to follow Poincaré's pace, ends in a nervous breakdown, which ruins his career of researcher. When Klein reproaches to Poincaré to have named 'Fuchsian' some of his newly discovered functions, to acknowledge the inspiration found in a memoir of Fuchs, the French mathematician reacts somewhat ironically by calling Kleinian the next class of functions he discovers !

Poincaré's motivation in this area comes from a question raised by Hermite for the *Grand Prix des Sciences Mathématiques de l'Académie des Sciences* of 1880 :

> To improve in some important point the theory of linear ordinary differential equations.

Poincaré's response to this question is a disordered sequence of submitted, withdrawn and supplemented memoirs, following the impetuous evolution of his thinking. This chaotic strategy disturbs the old lady of the *Quai Conti*, and the *Grand Prix* is awarded to Georges Halphen, for a more carefully written but less revolutionary memoir, with a first accessit only to Poincaré.

Besides non-Euclidian geometry, another basic ingredient of this first stream of research of Poincaré is Kronecker's index, inaugurating his use of topological tools in the study of singular points and limit cycles of differential equations, periodic solutions of the three body problem, and bifurcation of the equilibrium shapes of a rotating fluid when the speed of rotation increases.

# 5 The three stars of Hermite and King Oscar Prize

Besides Poincaré, two other rising stars also revolve around Hermite for, say family-mathematical reasons. The first one is Paul Appell, who has married a niece of Joseph Bertrand, brother-in-law of Hermite and most influencial mathematician and academician. The second one is Emile Picard, already famous in 1879 for his theorem on entire functions, and son-in-law of Hermite. The poor Hermite is submitted to the pressure of his wife, who supports Picard, and of his authoritary brother-in-law, who defends Appell. He writes to Mittag-Leffler, former student of Weierstrass and husband of the wealthy daughter of Finland's 'king of tobacco' :

> With a low voice and confidentially, fearing to be heard by Mrs Hermite, I tell you that from our three mathematical stars, Poincaré seems to me the most brillant one. Furthermore he is a charming young man, from Lorraine like me, who perfectly knows my family.

Sailing skilfully on the sea of vacant chairs at the *Sorbonne*, Hermite succeeds in appointing almost simultanuously Appell in mechanics, Picard in calculus and Poincaré in mathematical physics and probability. A similar game starts a few years later for their election at the *Académie des Sciences*. Poincaré is elected in 1887, Picard in 1889 and Appell in 1892.

In 1885, following a suggestion of Mittag-Leffler, King Oscar II of Sweden decides to celebrate his 60th birthday anniversary by a prize crowning an important discovery in mathematical analysis, an example unfortunately followed by few other monarchs. The award consists in a gold medal and 2.500 golden crowns. Any competing memoir must deal with one of the following topics:

1. The n-body problem in celestial mechanics.
2. Fuch's generalization of ultraelliptic functions.
3. The functions defined by a first order differential equation.
4. The algebraic relations between two fuchsian functions having a common group.

The competition perfectly fits Poincaré's mathematical interests, who decides to work on the first question. He sends in May 1888 a 160 pages memoir entitled *Sur le problème des trois corps et les équations de la dynamique.* Although the work does not completely answer the question, the committee, made of Weierstrass, Hermite and Mittag-Leffler, gives the award to Poincaré, adding that

> It is the deep and original work of a mathematical genious whose position is among the greatest mathematicians of the century. The most important and difficult questions, like the stability of the world system, are treated using methods which open a new era in celestial mechanics.

The French newspapers widely comment the event, and Poincaré is made *Chevalier de la Légion d'Honneur*.

During the printing of Poincaré's memoir, from July to November 1889, Phragmén, a young collaborator of Mittag-Leffler in charge of the editorial work, finds some parts mathematically unclear. The first explanations of Poincaré, concretized by nine added notes, are followed by a long silence. In a moving letter of December 1rst, 1889 Poincaré admits an error having important consequences : the conclusion of stability of the solar system is indeed invalid ! When the letter reaches Stockholm, the optimistic Mittag-Leffler has started the distribution of the issue of the *Acta Mathematica* containing the memoir, and he must use all his diplomacy and influence to get the issues back to Sweden. One of them is rediscovered in Stockholm in the last decade of the 20th century, in contradiction to a hand-written mention :

> The whole edition has been destroyed.

Poincaré finally sends a new version of the memoir in June 1890 – 270 pages long – and must pay its printing : more than the 2500 crowns of the Prize ! The malediction is not extinguished however : King Oscar's medal itself was stolen a few years ago in Poincaré's grand-son's appartment !

In correcting his mistake, Poincaré has discovered a gold mine for mathematics and science, by paving the way to chaos theory. In his own words :

> When one tries to depict the figure formed by these two curves and their infinity of intersections, each of which corresponds to a doubly asymptotic solution, these intersections form a kind of net, web or infinitely tight mesh. One is struck by the complexity of this figure that I am not even attempting to draw.

In one of his more popular writings, he later explains in a prophetic way the possible consequences of this discovery :

> It may happen that small differences in the initial conditions produce great ones in the final phenoma.

The butterfly effect is born, but hunting this butterfly has been for Poincaré quite a painful experience!

# 6 Mathematical physics

This extraordinary and turbulent period of research around King Oscar's Prize does not prevent Poincaré to take very seriously his teaching in the chair of mathematical physics. If not a great lecturer, he is a very conscientious one. Each semester, he chooses a new topics, and he prefaces and edits the notes taken by his best students. All in all are published more than a dozen of volumes, covering the whole classical physics (hydrodynamics, elasticity, potential theory, capillarity, thermodynamics, heat theory, optics, electromagnetism) and probability, where Poincaré reveals his inventiveness and mathematical virtuosity. Among other things, he carefully discusses Hertz experiments on the propagation of electromagnetic waves and the beginnings of wireless telegraphy.

His books on Maxwell theory contain the germs of special

relativity and lead him to analyze, correct and name the Lorentz transformations. Poincaré publishes in 1905 a note (followed by an extended memoir) on the dynamics of the electron, containing the whole mathematics of special relativity. The historians of science still

passionately discuss about the priority between Einstein and Poincaré, and, if one follows some recent publications, Hercule Poireau could be the only one able to uncover the whole story. Curiously, the mathematicien Poincaré reaches the relativistic kinematics via Maxwell electromagnetic theory, when the physicist Einstein uses an axiomatic method. But is is unquestionable that Poincaré has anticipated the so-called Minkowski space-time.

Poincaré also devotes three long memoirs between 1890 and 1895 to the partial differential equations of classical mathematical physics. He invents the sweeping method to solve the Dirichlet problem, proves for the first time the existence of infinitely many eigenvalues for the same problem, and introduces some inequalities which are still the cornerstones of the modern theory of partial differential equations.

One of the last scientific conferences attended by Poincaré is the first *Conseil Solvay*, in Brussels, from October 30 to November 3, 1911, held at the *Hôtel Métropole*. Lorentz, Poincaré, Planck, Marie Curie, Einstein, Perrin, Langevin, Rutherford and others are also there to discuss the last

developments of quantum theory. During this council, Poincaré insists on the main challenge of physics in this time, the construction of a coherent quantum theory :

> What striked me in the discussions that we have just heared, is to see the same theory to depend sometimes on the principles of the old mechanics, and sometimes on new hypothesis which are its negation; one should not forget that any proposition can be proved, as soon as one uses in the proof two contradictory statements.

Back to Paris, he publishes on this hot topics, in February 1912, one of his last papers, with shows the necessity of the quantum jumps in interpretating the experimental data.

With 49 proposals between 1901 and 1912, Poincaré is the most nominated scientist for the Nobel Prize in physics. The priority given to experimental physicists, Mittag-Leffler's enemies at the Swedish Academy of Science and the untimely death of Poincaré have prevented him to add the Nobel Prize to his amazing list of scientific awards.

# 7 Celestial mechanics and topology

After Tisserand's sudden death in 1896, Poincaré accepts, on request of his dean Darboux, the chair of theoretical astronomy and celestial mechanics. In academic affairs, Poincaré never behaves like a *prima donna,* and gives priority to the benefit of the institutions. Again, his lectures are published, one volume on the shapes of equilibrium of rotating fluids, a three volumes set on celestial

mechanics developing perturbation methods, lunar theory and a study of tides based on Fredholm integral equations, and a last volume on cosmogonical hypotheses. But his most famous publication in this area is the immortal *Méthodes nouvelles de la mécanique céleste,* published between 1892 and 1899, a widely expanded version of the memoir crowned by King Oscar's Prize.

The turn of the century also sees the publication of a series of six long memoirs on *Analysis situs,* i.e. on algebraic topology, where geometrical properties in arbitrary dimensions are deduced from those of associated

algebraic structures. The motivation comes from the study of nonlinear differential equations and of the three-body problem, but the theory is developed for its own sake, with applications to algebraic geometry. Between 1892 and 1901, Poincaré creates almost *ex nihilo* the basic tools of algebraic topology : fundamental homotopy group, simplicial homology, Euler-Poincaré's formula, duality principle. He even sketches De Rham cohomology, and, after proving that

> any compact and simply connected 2-dimensional manifold is homeomorphic to the usual sphere,

he states a famous conjecture :

> any compact and simply connected 3-dimensional manifold is homeomorphic to the 3-sphere.

This is to-day one of the seven famous 'one million dollars conjectures' of the Clay institute, and could have been proved very recently by the Russian mathematician Perelman. In another direction, Poincaré also initiates the modern theory of dimension, and, in a moving paper, the year of his death, he describes a fixed point theorem for continuous area-preserving mappings of an annulus which rotate the two boundaries in opposite directions. He knows his proof uncomplete, but fears not to have enough time left to fix it. This is done by George D. Birkhoff in 1913, and this area is nowadays a very active one in Hamiltonian dynamics and symplectic geometry.

# 8 Scientific philosophy and the Académie française

Besides technical works, Poincaré regularly publishes papers in popular science or philosophy journals. He discusses the role of logics in mathematics, the birth of set theory, the foundations of arithmetics, geometry and mechanics, the recent developments of physics. In 1902, the editor Flammarion convinces Poincaré to collect and edit this material for his famous series *Bibliothèque de Philosophie Scientifique.* The first volume, *La Science et l'Hypothèse,* is published in 1902, followed by *La Valeur de la Science* in 1905, and *Science et Méthode* in 1908. A posthumous volume *Dernières Pensées,* comes out in 1913, and an aborted fifth volume has been published last year.

The four orange covered books are often reprinted and are translated in many languages. Written in a witty style, they differ from the usual philosophical

language by a frequent sharp irony and a definite taste for paradox. Many philosophers have been abashed by a changing and self-critizing thought, which refuses to enclose the world in a single idea. Scientism is criticized by a first class scientist, who defends the idea of 'convenient' model, and can only be fully understood by readers fully aware of his scientific contributions.

Those popular books give to Poincaré an unwanted celebrity. In France, the secularization of the primary and secondary schools has generated a strong tension between catholics and radicals. While discussing the relativity of motion in mechanics, Poincaré writes :

> Absolute space, that is to say, the mark to which it would be necessary to refer the Earth to know whether it really moves, has no objective existence. The two propositions : 'The Earth turns round' and 'It is more convenient to suppose the Earth turns round' have the same meaning; there is nothing more in the one than in the other.

For catholic circles, this justifies Galileo's condemnation by the Church ! Needless to say that Poincaré has to loose time and energy to refute such a tendencious interpretation.

Following a long tradition, the *Académie française* always elects a few scientists, who have added style to their scientific discoveries. They indeed are mostly helpful in writing the definitions of scientific words for the dictionary that the learned assembly publishes. d'Alembert, Condorcet, Laplace, Fourier, Bertrand, Poincaré and Picard have became *Immortels*. Since Picard's death in 1941, no mathematician has got this honor, showing that our corporation should care more about style.

Henri Poincaré is elected in 1908, at the seat number 24 occupied by Sully-Prudhomme. Following the tradition, he must make the eulogy of the poet at his reception to the Academy, January 28, 1909. The story is pleasantly told in a chronicle of André Beaunier :

> He had, like a mundaine writer, like a playwright, attracted the crowd. Algebra maybe was the new fashion this winter. The greatest living mathematician did not read badly his speach. ¿From time to time, he gave the feeling that he was thinking to something else, but when remembering his present adventure, he took a more familiar accent. When a page was read, he looked quite happy, and throwed it rapidly behind him. At the end of his speach, he sat on the pile with a great satisfaction.

Sully-Prudomme's eulogy is reproduced in the delicious but little known small volume *Savants et écrivains,* whose introduction remains the best description of Poincaré's idea of the activity of a scientist. Sully-Prudhomme, was born in Paris in 1839. First attracted by science, but prevented to enter the *Ecole Polytechnique* by an illness of the eyes, Sully-Prudhomme became batchelor in letters. After translating Lucrece, he dreamed of unifying poetry and

science, and wrote long philosophical poems, which made him, in 1901, the first Nobel Prize in literature. In a sense Poincaré was lucky, as he observes himself :

> One will perhaps be surprised to learn that Sully-Prudhomme has left a long manuscript on the philosophy of mathematics; it looks like he was trying from the beginning to justify as much a possible my presence here.

Instead of taking advantange of this unexpected help, Poincaré carefully analyzes Sully-Prudhomme's poetry and philosophy, concluding in a way which underlines his own conception of philosophy :

> But I must stop, there is in the philosophical vocabulary too many words ending in 'iste' and this infinite multitude scares me. [...] The soul of a true philosopher is a battlefield; this is not a peaceful monarchy where there is place for one master only.

# 9 Poincaré and public affairs

The famous Dreyfus affair gives Poincaré another opportunity to leave his ivory tower. In 1894, the French Intelligence Service finds a memorandum sent to the German military attaché in Paris, anouncing the sending of confidential information. An apparent similarity in hand writing leads to the arrest of a French officier of Jewish origin, Alfred Dreyfus. He is declared guilty by the court-martial in 1895 and deported to the *Ile du Diable* in Guyane. France is soon divided into supporters and opponents to Dreyfus. After a long and passionate fight, a revision of the first trial takes place in Rennes in 1899. The famous police expert

Bertillon had used pseudo-scientific techniques and probability theory in his graphological analysis of the memorandum. He concludes as follows his testimony :

> In the collection of observations and agreements which constitute my demonstration, there is no place for doubt; and this is with this not only theoretical but material certitude, that with the feeling of responsability following from such an absolute certitude, in all honesty, I affirm, to-day like in 1894, under oath, that the memorandum is the work of the defendant.

Such a statement is philosophically unbearable to Poincaré. In a letter written at the request of Painlevé and read to the Court, he violently reacts again the use of probability theory in Bertillon's conclusions :

> Nothing in that has any scientific character. I do not know if the defendant will be sentenced, but if he is, it must be on other evidences. It is impossible that such an argument makes any impression on free minded people having received a solid scientific education.

But the Court-martial declares again Dreyfus guilty, this time with mitigating circumstances. Dreyfus gets the presidential pardon, but a new revision is finally obtained by Dreyfus supporters in 1904. Poincaré concludes as follows a long report, jointly written with Appell and Darboux :

> All those systems are absolutely deprived of any scientific value :
>
> 1. because the application of probability theory to those questions is not legitimated;
> 2. because the reconstruction of the memorandum is false;
> 3. because the rules of the probability theory have not been correctly applied.
>
> In a word, because the authors have wrongly argued on false documents.

The conclusions of this report take a full page in the twenty ones of reasons adduced to the Court's sentency, declaring Dreyfus innocent and restoring his honor and his rights.

In contrast with other contemporary scientists, like Painlevé, Hadamard, Borel, Perrin or Langevin, Poincaré always refuses any political engagement or duty. In 1904, in response to an inquiry of the *Revue bleue*, he expresses his feelings about politics in his usual ironic style :

> Politics is nowadays a profession which entirely absorbs a man; any scientist who wants to dedicate himself to politics must give up his vocation; if he really wants to be useful to the country, he must give half of his time to the affairs of the Republic; if he wants to keep his seat, he must give the other half to the affairs of his electors; nothing will be left for science. It would therefore be inopportune that all scientists would aim at the Parliament, because, then, there would be no scientists anymore. One can be resigned, or even rejoice not only for the country, but for science itself, to sacrifice from time to time one of us, more able to be understood by the multitude or the assemblies. After all science needs somebody to defend its interests.

But Poincaré never refuses any engagement or duty in the administration and organisation of science. The list of his responsabilities in this respect covers two long pages. The astronomer J. Levy has expressed his regrets of the time lost by Poincaré in those activities, specially in the last ten years of his life :

> Maybe should one regret that, from this time, the more and more heavy duties he accepts conscientiously do not give him anymore the time to polish his works. He gives a part of the best of himself to the numerous academies, associations, councils and committees who have sollicited the favour to receive him. He consumes himself in tasks which do not fit his measure. For example, as chairman of the Commission devoted to organize the revision of the arc of meridian in Quito, he writes himself, from 1900 to 1905, all the corresponding reports. In 1900, he discusses the savings which can be done when buying the she-mules; in 1902, the measures to be taken to remedy to the destruction of the geodesic signals by the Indians; in 1905, the reproduction in coloured pictures of the insects found by the expedition.

His repute, and the attraction that the events in the sky make on the multitude, constantly attract to him journalists short of copy. The exceptionally rainy year 1910 having been linked in the media to the passage of a comet, Poincaré reacts with humour, refering to the tradition connecting the production of good wine, and not of water, to the presence of a comet !

## 10 Poincaré and science

Some superficial readers confuse Poincaré's philosophical ideas about science, his so-called conventionalism, with a scepticism for science. All we have said about his activity contradicts such an assertion and proves the deep feelings and the entire devotion of the French mathematician to scientific activity. As Emile Borel writes in 1954 :

> Some people have considered Poincaré as a sceptic, other ones like the forerunner of axiomatic methods; but he would have refused to be brigaged in any sect, even if this sect could refer to his thought. For him, the moral of the scientist can be summarized in a law that the mere moral condemns : the aim justifies the means. The aim is the knowledge of the Universe, it is the agreement between the numerical results deduced from the formulas and the numbers written by the physicists and the astronomers in their observation books. The means, for the mathematician, are the formulas and a language he has the right to create at his own convenience.

Poincaré has expressed strong feelings about the freedom of science. In 1909, promoted doctor *honoris causa* at the 75th anniversary of the *Université Libre de Bruxelles,* he says :

> Freedom is for Science what air is for animal; deprived of this freedom, it dies from suffocation, like a bird deprived of oxygen. And this freedom must be without limits, because, if one wants to impose some, one gets a half-science only, and half-science is no more science, because it can be, it necessarily is, a false science. The thought must never be subordinated to any dogma, political party, passion, interest, preconceived idea, to anything indeed, except the facts themselves, because, for science, to be subordinated means to die.

The last sentence is reproduced on the walls of the main building of the University of Brussels.

Poincaré often insists on the aesthetical motivations of scientific activity :

> The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature was not beautiful, it would not be worth knowing, and if nature was not worth knowing, life would not be worth living. Of course I do not speak here of that beauty that strikes the senses,

> the beauty of quality and appearances; not that I undervalue such beauty, far from it, but it has nothing to do with science; I mean that deeper beauty coming from the harmonious order of the parts, and that a pure intelligence can grasp.

Some of the mathematical considerations of Poincaré have inspired various trends in modern art.

The Polytechnician Poincaré, who contributed so much to the applications of science, unambiguously insists on the essential long term investment of fundamental research :

> The scientist must not loiter in realising practical aims. He without doubts will obtain them, but must obtain them in addition. He never must forget that the special object he is studying is only a part of this big whole, which must be the sole motive of his activity. Science has had marvellous applications, but a science which would only have applications in mind would not be science anymore, it would be only cookery.

Those words remain more than ever of crucial importance. Victim of the increasing resignation of the public authority, science is more and more threatened, in its freedom, by economical forces dominated by immediate profit, and fundamental research is constantly identified, in the minds of decision-makers and in the public agency budgets, with development.

# 11 Conclusion

When Poincaré suddenly dies July 17th, 1912, from an embolism consecutive to a surgery, the scientific world is far from ready to benefit from his legacy. According to the great French mathematician Jean Leray :

> Very few men were able to follow his thoughts; he had no students. After one century of mathematical work, we can understand his thoughts more easily, speak about them in a more familiar way; but the more we approach them, the more we admire and respect them.

Another great French mathematician, André Weil, insists on the actuality of Poincaré's work :

> On this point like on many other ones, I hope to have shown you that Poincaré's work belongs not only to the history of our science; it also belongs to the most living actuality.

I leave the last words – *nationalisme oblige* – to the famous mathematical physicist David Ruelle :

> Mathematical physics tries to understand a world of unknown complexity with tools of known limitation. This requires boldness, and modesty. Obviously Henri Poincaré lacked neither of these two qualities.

The following lectures will masterly confirm those eminent opinions.

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