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Equidistribution problems for line bundles on projective manifolds

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Abstract

Let L be a holomorphic line bundle on a projective manifold X and $H^0(X, L^p)$ be the space of global holomorphic sections of $L^p := L^{\otimes p}$. Suppose that h is a smooth Hermitian metric on L with positive curvature $c_1(L, h) > 0$. A foundational result of Tian states that certain Fubini-Study forms associated to the spaces $H^0(X, L^p)$, where L^p is endowed with the product metric $h^{\otimes p}$, converge as $p \to \infty$ to $c_1(L, h)$ in the $\mathcal{C}^\infty$ topology on X. Using this, Shiffman and Zelditch showed that, if the spaces $H^0(X, L^p)$ are endowed with Gaussian probability measures, then for almost every sequence of sections $\{S_p \in H^0(X, L^p)\}_{p \geq 1}$ the zero divisors $\frac{1}{p}[S_p = 0] \to c_1(L, h)$ as $p \to \infty$, in the weak sense of currents on X.

We discuss generalizations of these results to the case when the metric h on L is singular and the spaces $H^0(X, L^p)$ are endowed with quite general probability measures. These are based on joint works with Xiaonan Ma and George Marinescu, and with Turgay Bayraktar and George Marinescu.