

The level sets of functions with bounded critical sets and bounded Hess⁺ complements

Cornel Pinte

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

We denote by Hess⁺(f) the set of all points $p \in \mathbb{R}^n$ such that the Hessian matrix $H_p(f)$ of the C^2 -smooth function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is positive definite. In this paper we prove several properties of real-valued functions of several variables by showing the connectedness of their level sets for sufficiently high levels, under the boundedness assumption on the critical set. In the case of three variables we also prove the convexity of the levels surfaces for sufficiently high levels, under the additional boundedness assumption on the Hess⁺ complement. The selection of the *a priori* convex levels, among the connected regular ones, is done through the positivity of the Gauss curvature function which ensure an ovaloidal shape of the levels to be selected. The ovaloidal shape of a level set makes a diffeomorphism out of the associated Gauss map. This outcome Gauss map diffeomorphism is then extended to a smooth homeomorphism which is used afterwards to construct one-parameter families of smooth homeomorphisms of Loewner chain flavor.