Workshop dedicated to the memory of Professor Gabriela Kohr
(2nd edition)
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

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Filtration of generators and an inverse Fekete–Szegö problem

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Abstract

n this talk we present problems belonging to (a) dynamic system and to (b) geometric function theory, in their correlation. In the first part of the talk, we study the problem of characterizing membership of normalized holomorphic functions of the disk to the class of infinitesimal generators and some its subclasses as well as dynamical properties of generated semigroups. Presenting results include analytic extension in the semigroup parameter and the uniform convergence. Our approach is based on so-called 'filtrations' of the class of infinitesimal generators.

In the second part we introduce and study a question that can be interpreted as 'an inverse Fekete–Szegö problem'. This problem links to the first part of the talk. We introduce new filtration classes using the non-linear differential operator

$$\alpha \cdot \frac{f(z)}{z} + \beta \cdot \frac{zf'(z)}{f(z)} + (1-\alpha-\beta) \cdot \left[1 + \frac{zf''(z)}{f'(z)}\right],$$

and establish certain properties of these classes. Sharp upper bounds of the absolute value of the Fekete–Szegö functional over some filtration classes are found. We also present open problems for further study.

The talk is based on joint works [1, 2, 3].

References

[1] F. Bracci, M. D. Contreras, S. Díaz-Madrigal, M. Elin and D. Shoikhet, *Filtrations of infinitesimal generators*, Funct. Approx. Comment. Math. **59** (2018).

[2] M. Elin, D. Shoikhet, and T. Sugawa, *Filtration of semi-complete vector fields revisited*, in: Trends in Math., Birkhäuser/Springer, Cham, 2018.

[3] M. Elin, F. Jacobzon and N. Tuneski, *The Fekete–Szegö problem and filtration of generators*, Rendiconti del Circolo Matematico di Palermo Series 2, DOI 10.1007/s12215-022-00824-w.