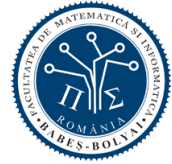




Faculty of Mathematics and Computer Science
Babeş-Bolyai University of Cluj-Napoca



Workshop dedicated to the memory of
Professor Gabriela Kohr
(2nd edition)

Geometric Function Theory in Higher Dimensions and Complex Banach Spaces

Book of Abstracts

1 – 3 December 2022
Cluj-Napoca, Romania

Invited Speakers

- **Lucian Beznea** - Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania
- **Filippo Bracci** - Tor Vergata University, Rome, Italy
- **Teodor Bulboacă** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Martin Chuaqui Farrú** - Pontifical Catholic University of Chile, Santiago, Chile
- **Dan Coman** - Syracuse University, New York, USA
- **Mihai Cristea** - University of Bucharest, Romania
- **Paula Curt** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Tamás Darvas** - University of Maryland, USA
- **Ciprian Demeter** - Indiana University, Bloomington, USA
- **Santiago Diaz-Madrigo** - University of Seville, Spain
- **Mark Elin** - Braude College, Karmiel, Israel
- **Anatoly Golberg** - Holon Institute of Technology, Holon, Israel
- **Ian Graham** - University of Toronto, Canada
- **Pavel Gumenyuk** - Polytechnical University of Milan, Italy
- **Hidetaka Hamada** - Kyushu Sangyo University, Fukuoka, Japan
- **Tatsuhiko Honda** - Senshu University, Tokyo, Japan
- **Mihai Iancu** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Fiana Jacobzon** - Braude College, Karmiel, Israel
- **Cezar Joița** - Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania
- **David Kalaj** - University of Montenegro, Podgorica, Montenegro
- **Massimo Lanza de Cristoforis** - University of Padova, Italy

- **Sergey Mikhailov** - Brunel University London, UK
- **Victor Nistor** - Universite Lorraine, Metz, France
- **Mihai Pascu** - Transilvania University of Braşov, Romania
- **Cornel Pinte**a - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Radu Precup** - Babeş-Bolyai University, Cluj-Napoca, Romania
- **Oliver Roth** - University of Wuerzburg, Germany
- **David Shoikhet** - Holon Institute of Technology & Braude College, Israel
- **Matti Vuorinen** - University of Turku, Finland
- **Wolfgang L. Wendland** - University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH, Germany

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Classical and recent results on the existence of invariant measures for transition functions

Lucian Beznea

Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

First, we shall review the classical Krylov-Bogolyubov result and the Doeblin condition for the existence of invariant (equilibrium) distributions of a Markov process. Then we present some recent developments on this topic, obtained jointly with Iulian Cîmpean (Bucharest) and Michael Röckner (Bielefeld).

Norm estimates for some subclasses of Ma-Minda type functions

Teodor Bulboacă

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

For certain subclasses of analytic function of Ma-Minda type functions defined by differential subordination we determine Pre-Schwarzian norm estimate and inclusion criteria. Also, using the Gronwall's inequality we give a sufficient condition for a normalized function to belong to a class of functions with bounded arguments that extends the class of strongly α -Bazilevič functions of order γ studied in 1996 by Gao.

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2. Gao, C. Fekete-Szegő problem for strongly Bazilevič functions. *Northeast Math. J.* **1996**, *12*, 469–474.
3. Kim, J.A.; Sugawa T. Geometric properties of functions with small Schwarzian derivatives. (preprint) Available online: <http://www.cajpn.org/pp04/0403.pdf>.
4. Kim Y.C.; Sugawa, T. Norm estimates of the pre-Schwarzian derivatives for certain classes of univalent functions. *Proc. Edinb. Math. Soc.* **2006**, *49*, 131–143.
5. Ma, W.C.; Minda, D. A unified treatment of some special classes of univalent functions, In Proceedings of the Conference on Complex Analysis, Tianjin, China, 19–23 June 1992; Li, Z., Ren, F., Yang, L., Zhang, S., Eds.; International Press: Cambridge, MA, USA, 1994; pp. 157–169.

An overview of the Schwarzian derivative in several complex variables

Martin Chuaqui Farrú
Pontifical Catholic University of Chile, Santiago, Chile

Abstract

We present a survey of one of the definitions of Schwarzian derivatives in several complex variables that is due to T. Oda. We discuss the main properties such as invariance, a chain rule and a connection with a system of partial differential equations that involve the jacobian of the mapping. The system provides a formal characterization of univalence analogous to the case in one variable. We will also discuss how these operators give rise to differential operators of order two and three, and how these operators inherit a norm from the domain. We analyze the important procedure of normalization at a fixed point in a given domain, and show how this relates to the order of the families with bounded Schwarzian norm. As a consequence of the fact that in the ball these norms are invariant under the group of automorphisms, the normalization at the origin carries through to a normalization everywhere, and from there to an analogue of the Ahlfors-Weill extension for mappings with sufficiently small Schwarzian norm. This involves joint work with Rodrigo Hernández.

Extension of quasisubharmonic functions

Dan Coman
Syracuse University, New York, USA

Abstract

Let (V, ω) be a compact Kähler manifold and X be an analytic subvariety of V . We address the problem of extending ω -plurisubharmonic functions on X to ω -plurisubharmonic functions on V . Our results are joint with Vincent Guedj and Ahmed Zeriahi.

**On the radial limits of mappings satisfying the inverse Poletsky
modular inequality on Riemannian manifolds**

Mihai Cristea
University of Bucharest, Romania

Abstract

We study the geometric properties of the mappings for which a generalized inverse Poletsky modular inequality holds. Our approach is on Riemannian manifolds and we generalize some known theorems from the theory of analytic mappings concerning radial limits, such as the theorems of Fatou and M. and F. Riesz and their extensions given for quasiregular mappings by Martio and Rickman.

Janowski subclasses of biholomorphic mappings

Paula Curt

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

We survey some recent results concerning Janowski starlikeness and almost starlikeness in several complex variables. We also introduce and study some new subclasses of biholomorphic mappings of Janowski type.

The Mabuchi geometry of low energy classes

Tamás Darvas
University of Maryland, USA

Abstract

Let (X, ω) be a Kähler manifold and $\psi : \mathbb{R} \rightarrow \mathbb{R}_+$ be a concave weight. We show that the space of smooth Kähler potentials admits a natural metric d_ψ whose completion is the low energy space \mathcal{E}_ψ , introduced by Guedj-Zeriahi. As d_ψ is not induced by a Finsler metric, the main difficulty is to show that the triangle inequality holds.

Restriction of exponential sums to hypersurfaces

Ciprian Demeter
Indiana University, Bloomington, USA

Abstract

The last decade has witnessed a revolution in the circle of problems concerned with proving sharp moment inequalities for exponential sums on tori. This has in turn led to a better understanding of pointwise estimates, but this topic remains extremely challenging. One way to bridge the gap between global and pointwise behavior is to study the restriction of exponential sums to submanifolds of the torus. In my talk I will explore the behavior of lower dimensional Weyl sums and toral Laplace eigenfunctions, restricted to hypersurfaces. The proofs will involve decoupling, a Fourier analytic tool that has recently found a broad range of applications in many areas of mathematics.

Slope problems in the theory of semigroups of holomorphic self-maps of the unit disc

Santiago Diaz-Madrigal
University of Seville, Spain

Abstract

Given a semigroup (φ_t) of holomorphic self-maps of the unit disc D and fixed a point $z \in D$, the function $t \in [0, +\infty) \mapsto \varphi_t(z)$ can be seen as the trajectory of a certain vector field. Indeed, many times these trajectories land at concrete points in the circle. From a dynamical point of view, this suggests the question of when a definite slope is well defined for those trajectories or, more generally, to analyse all different dynamical behaviour which can happen. In this talk, we give a panoramic view of this problem for the so called parabolic semi- groups, paying special attention to several very recent developments in the area.

Filtration of generators and an inverse Fekete–Szegő problem

Mark Elin

Braude College, Karmiel, Israel

Abstract

In this talk we present problems belonging to (a) dynamic system and to (b) geometric function theory, in their correlation. In the first part of the talk, we study the problem of characterizing membership of normalized holomorphic functions of the disk to the class of infinitesimal generators and some its subclasses as well as dynamical properties of generated semigroups. Presenting results include analytic extension in the semigroup parameter and the uniform convergence. Our approach is based on so-called ‘filtrations’ of the class of infinitesimal generators.

In the second part we introduce and study a question that can be interpreted as ‘an inverse Fekete–Szegő problem’. This problem links to the first part of the talk. We introduce new filtration classes using the non-linear differential operator

$$\alpha \cdot \frac{f(z)}{z} + \beta \cdot \frac{zf'(z)}{f(z)} + (1 - \alpha - \beta) \cdot \left[1 + \frac{zf''(z)}{f'(z)} \right],$$

and establish certain properties of these classes. Sharp upper bounds of the absolute value of the Fekete–Szegő functional over some filtration classes are found. We also present open problems for further study.

The talk is based on joint works [1, 2, 3].

References

- [1] F. Bracci, M. D. Contreras, S. Díaz-Madrigal, M. Elin and D. Shoikhet, *Filtrations of infinitesimal generators*, *Funct. Approx. Comment. Math.* **59** (2018).
- [2] M. Elin, D. Shoikhet, and T. Sugawa, *Filtration of semi-complete vector fields revisited*, in: *Trends in Math.*, Birkhäuser/Springer, Cham, 2018.
- [3] M. Elin, F. Jacobzon and N. Tuneski, *The Fekete–Szegő problem and filtration of generators*, *Rendiconti del Circolo Matematico di Palermo Series 2*, DOI 10.1007/s12215-022-00824-w.

Metric approach in geometric function theory

Anatoly Golberg
Holon Institute of Technology, Israel

Abstract

In this lecture we study the main relationships between various classes of mappings whose definitions rely on metric approaches and techniques: finitely bi-Lipschitz, quasisymmetric, quasimöbius and quasiconformal mappings and mappings of finite metric and of finite area distortions. The latter are the central objects in our presentation. Although no analytic restrictions are assumed, some nice and important regularity properties (like absolute continuity on measure) are derived. We also involve classes of mappings which are called the ring, lower and hyper Q -homeomorphisms and are defined purely geometrically. The interplay between the above classes of mappings allows us to investigate the boundary correspondence problems related to the weakly flat and strongly accessible boundaries on Riemannian manifolds. Several illustrated examples are also presented.

Iteration, Loewner Theory, and Markov Processes

Pavel Gumenyuk
Polytechnical University of Milan, Italy

Abstract

In this talk we will review classical and new applications of Complex Analysis to Markov stochastic processes in which the transition probabilities turn out to be encoded by holomorphic self-maps and the Chapman-Kolmogorov relation can be written in terms of their compositions. In particular, there will be presented recent results for time-inhomogeneous continuous-state branching processes obtained in collaboration with Takahiro Hasebe (Hokkaido University, Sapporo, Japan) and José-Luis Pérez (Centro de Investigación en Matemáticas, Guanajuato, Mexico).

***A*-normalized univalent subordination chains and Loewner PDE in infinite dimensions**

Hidetaka Hamada
Kyushu Sangyo University, Fukuoka, Japan

Abstract

In this talk, we obtain the biholomorphicity of the univalent Schwarz mappings $v(z, s, t)$ with normalization $Dv(0, s, t) = e^{-(t-s)A}$ for $t \geq s \geq 0$, where $m(A) > 0$, which satisfy the semigroup property on the unit ball of a complex Banach space X . We further obtain the biholomorphicity of A -normalized univalent subordination chains under some normality condition on the unit ball of a reflexive complex Banach space X . We give the existence of the biholomorphic solutions $f(z, t)$ of the Loewner PDE with normalization $Df(0, t) = e^{tA}$ on the unit ball of a separable reflexive complex Banach space X . The results obtained in this talk give some positive answers to the open problems and conjectures given by Graham, Hamada, Kohr and Kohr in 2013. This is a joint work with Ian Graham, Gabriela Kohr and Mirela Kohr.

Composition operators between Bloch-type spaces on the homogeneous unit balls

Tatsuhiro Honda
Senshu University, Tokyo, Japan

Abstract

From the point of view of the Riemann mapping theorem, a homogeneous unit ball of a complex Banach space is a natural generalization of the open unit disc in \mathbb{C} . Every bounded symmetric domain in a complex Banach space is biholomorphically equivalent to a homogeneous unit ball. A complex Banach space X is a JB*-triple iff the open unit ball of X is homogeneous. Let \mathbb{B}_X be a bounded symmetric domain realized as the open unit ball \mathbb{B}_X of a finite dimensional JB*-triple X . In this talk, we discuss about the composition operator C_φ between Bloch-type spaces, where φ is a holomorphic mapping from \mathbb{B}_X into \mathbb{B}_Y .

This is a joint work with Hidetaka Hamada.

On Loewner chains and related problems for bounded balanced pseudoconvex domains in \mathbb{C}^n

Mihai Iancu

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

Let $\Omega \subset \mathbb{C}^n$ be a bounded balanced pseudoconvex domain with C^1 plurisubharmonic defining functions. First, we present a proof of the compactness of the Carathéodory family $\mathcal{M}(\Omega)$ with respect to the topology of locally uniform convergence on $H(\Omega)$. Next, we survey some results in the theory of Loewner chains on Ω , which are extensions of corresponding results on the Euclidean unit ball.

This talk is based on joint work with Hidetaka Hamada and Gabriela Kohr.

The Fekete–Szegő problem for spirallike mappings and non-linear resolvents in Banach spaces

Fiana Jacobzon
Braude College, Karmiel, Israel

Abstract

We study the Fekete–Szegő problem on the open unit ball of a complex Banach space. Namely, the Fekete–Szegő inequalities are proved for the class of spirallike mappings relative to an arbitrary strongly accretive operator, and some of its subclasses. Next, we consider families of non-linear resolvents for holomorphically accretive mappings vanishing at the origin. We solve the Fekete–Szegő problem over these families.

Based on the join work with Mark Elin.

Projection of connected Stein open subsets via surjective holomorphic maps

Cezar Joița

Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

Suppose that X and Y are connected reduced complex spaces and $f : X \rightarrow Y$ is an open surjective holomorphic map. We prove that for every connected open subset U of Y there exists a connected Stein open D subset of X such that $f(D) = U$. Joint work with Mihnea Coltoiu.

Gaussian curvature conjecture

David Kalaj

University of Montenegro, Podgorica, Montenegro

Abstract

In this paper we solve the longstanding Gaussian curvature conjecture of a minimal graph S over the unit disk. This conjecture states the following. For any minimal graph lying above the entire unit disk, the Gaussian curvature at the point above the origin satisfies the sharp inequality $|\mathcal{K}| < \pi^2/2$. The conjecture is first reduced to the estimation of the Gaussian curvature of certain Scherk type minimal surfaces over some bicentric quadrilaterals inscribed in the unit disk containing the origin. Then we make a sharp estimate of the Gaussian curvature of those minimal surfaces over those bicentric quadrilaterals at the point above the zero. Our proof uses complex-analytic methods since minimal surfaces that we consider allow conformal harmonic parameterization.

On the boundary behaviour of the double layer potential of second order elliptic differential operators

Massimo Lanza de Cristoforis
University of Padova, Italy

Abstract

The boundary integral operator corresponding to the double layer potential on the boundary of some smooth open subset of the Euclidean space is well known to improve the regularity of the functions. If the boundary is smooth the improvement is of one unit of regularity. We present some generalizations of known results in case the regularity of the set is C^2 or lower in the frame of Schauder spaces.

On Spatially-Periodic Solutions of Evolution Anisotropic Stokes, Oseen, and Navier-Stokes Equations

Sergey E. Mikhailov
Brunel University London, UK

Abstract

First, we show that the evolution (non-stationary) anisotropic Stokes and generalised Oseen systems with variable viscosity coefficients in a compressible framework can be classified as parabolic systems in the sense of Solonnikov. The solution uniqueness, existence and regularity of these linear PDE systems are analysed in a range of periodic Sobolev (Bessel-potential) spaces in \mathbb{R}^n . Then, employing the Galerkin algorithm, we prove the existence of a global weak solution for the evolution anisotropic variable-coefficient non-linear Navier-Stokes system in a periodic Sobolev space. The solution uniqueness and regularity results are also discussed.

Layer potentials on manifolds with cylindrical ends

Victor Nistor

Université de Lorraine, Metz, France

Abstract

We study the method of Layer Potentials on manifolds with cylindrical ends. This includes domains in \mathbb{R}^n with outlets at infinity. One of the main difficulties is the characterization of the Fredholm properties of the resulting integral operators, which requires information on the behavior at infinity. Joint work with Marius Mitrea, Mirela Kohr and Wolfgang L. Wendland. We apply our results to the study of the Laplacian and of the Stokes system.

Sequence ordering and applications

Mihai Pascu

Transilvania University of Braşov, Romania

Abstract

On the set of (finite) sequences of real numbers, we introduce a natural ordering relation, closely connected to the stochastic ordering of random variables. We present some of the properties of this ordering, and we derive some of its applications (a general Cauchy condensation theorem, convex ordering of Pólya random variables, etc).

We conclude with some extensions to the case of sequences of complex numbers (work in progress).

The talk is based on joint work with Nicolae R. Pascu (Kennesaw State University, USA) and Antonia Ioana Ţacă (Transilvania University of Braşov, Ph.D. student).

The level sets of functions with bounded critical sets and bounded Hess⁺ complements

Cornel Pinte

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

We denote by $\text{Hess}^+(f)$ the set of all points $p \in \mathbb{R}^n$ such that the Hessian matrix $H_p(f)$ of the C^2 -smooth function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is positive definite. In this paper we prove several properties of real-valued functions of several variables by showing the connectedness of their level sets for sufficiently high levels, under the boundedness assumption on the critical set. In the case of three variables we also prove the convexity of the levels surfaces for sufficiently high levels, under the additional boundedness assumption on the Hess⁺ complement. The selection of the *a priori* convex levels, among the connected regular ones, is done through the positivity of the Gauss curvature function which ensure an ovaloidal shape of the levels to be selected. The ovaloidal shape of a level set makes a diffeomorphism out of the associated Gauss map. This outcome Gauss map diffeomorphism is then extended to a smooth homeomorphism which is used afterwards to construct one-parameter families of smooth homeomorphisms of Loewner chain flavor.

Analysis of Navier-Stokes models for flows in bidisperse porous media

Radu Precup

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

Having in view a model proposed by Nield and Kuznetsov (2005, 2013), we consider a more general system of coupled Navier-Stokes equations in the incompressible case subject to the homogeneous Dirichlet condition in a bounded domain. Due to its generality, this system can be regarded as a new model for fluid flows in bidisperse porous media. We provide a deep theoretical analysis for large classes of equations and coupled systems of Navier-Stokes type with various non-homogeneous terms of reaction type. Existence results are obtained by using a variational approach making use of several fixed point principles and matrix theory.

Geometric versions of Schwarz's lemma for spherically convex functions

Oliver Roth
University of Wuerzburg, Germany

Abstract

We discuss several sharp distortion and monotonicity theorems for spherically convex functions defined on the unit disk involving geometric quantities such as spherical length, spherical area and total spherical curvature. These results can be viewed as geometric variants of the classical Schwarz lemma for spherically convex functions.

Joint work with Maria Kourou.

Fixed point theory of holomorphic mappings in complex spaces and applications

David Shoikhet

Holon Institute of Technology & Braude College, Israel

Abstract

This talk is devoted to the recent developments in the fixed point theory of holomorphic and hyperbolically non-expansive mappings in complex spaces.

The results presented here mostly are based on works of Marco Abate, Vladimir Bolotnikov, Filippo Bracci, Clifford Earle, Mark Elin, Lawrence Harris, Kazimierz Goebel, Ien Graham, Richard S. Hamilton, Gabriela Kohr, Tadeush Kuchzumow, Simeon Reich, David Shoikhet, Adam Stachura, Toshiyuki Sugawa, Lawrence Zalcman and others.

Also, we would like to mention a recent great contribution to the theory of fixed points of holomorphic mappings and semigroups in the spirit of complex dynamical systems developed by Leonardo Arosio, Manuel D. Contreras and Santiago Diaz-Madrigal.

Actually, the most powerful tool in those studies is the so-called *resolvent family* of a discrete (or continuous) semigroup of holomorphic mappings, which is based particularly on the previous results given by M.Elin, D.Shoikhet and T.Sugawa.

Since the class of nonlinear resolvents consists of univalent functions, it can be studied in the frameworks of classical and modern geometric function theories.

The infinite dimensional theory of holomorphic functions originated in a series of papers by M. Fréchet and R. Gateaux that appeared from 1909 to 1929 and was subsequently developed by many others. We first discuss two definitions of holomorphy. A strong definition is due to Fréchet and a weak definition usually is associated with Gateaux.

Conformal Geometry, Capacity and Computation

Matti Vuorinen
University of Turku, Finland

Abstract

This talk is a survey of my latest joint research on computation of conformal capacity, see arXiv 2021-2022. In particular we study the capacity of a family of disjoint hyperbolic disks. The work is based on computational methods and we discuss also several open problems of the field. Two numerical methods are used: fast boundary integral equation method of M.M.S. Nasser and hp -FEM method implementation of H. Hakula.

On the computation of transmission problems

Wolfgang L. Wendland

University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH,
Germany

Abstract

In the works [1] and [2] it is proven, that transmission problems for the anisotropic Stokes systems in Lipschitz domains with transversal interfaces have unique solutions in appropriate Hilbert and Sobolev spaces. In this lecture we consider the construction of harmonic solutions to boundary value problems in Lipschitz domains with transversal interfaces which can be constructed with the help of screen problems under comparability conditions which exclude the singular parts of the screen problems.

1. M. Kohr, S.E. Mikhailow, W.L. Wendland: *Non-homogeneous Dirichlet–transmission problems for the anisotropic Stokes and Navier–Stokes systems in Lipschitz domains with transversal interfaces*, Calculus of Variations and Partial Differential Equations **61** (2022) N06, Nb198, 47 pages.
2. M. Kohr, S.E. Mikhailow, W.L. Wendland: *On some mixed transmission problems for the anisotropic Stokes and Navier–Stokes systems in Lipschitz domains with transversal interfaces*. Journal of Mathematical Analysis and Applications **516** (2022) 126464, 28 pages.

Contributed talk

The Graham-Kohr extension operator and subclasses of biholomorphic mappings in \mathbb{C}^n

Eduard S. Grigoriuc
Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

Let $\mathbb{B}^n \subseteq \mathbb{C}^n$ be the Euclidean unit ball. We present two subclasses of convex, respectively starlike biholomorphic mappings on \mathbb{B}^n together with some interesting properties of them. Using the Graham-Kohr extension operator (introduced by I. Graham and G. Kohr in 2002) we study how the previous subclasses are preserved from one to several complex variables.