Workshop dedicated to the memory of Professor Gabriela Kohr
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

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First order elliptic systems in the plane

Wolfgang L. Wendland
University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH, Germany

Abstract

The lecture is based on [Elliptic Systems in the Plane, Pitman, London 1979] and is concerned with first order real systems of n linear equations for vector-valued functions $\mathbf{u} \subset \mathbb{R}^n$, elliptic in the sense of Petrovski. In the simplest case and $n = 2$, these are the Cauchy–Riemann equations for holomorphic functions w in $D \subset \mathbb{C}$. For Riemann–Hilbert boundary value problems in the unit disc D with boundary conditions $Re(\Lambda w)|_{|z|=1} = \varphi$, for w in the Sobolev space $W_2^{\varrho}(D)$ Fritz Noether's theorem is valid and the Fredholm index is $\kappa = 1 - 2\omega = \alpha - \beta$ where $\alpha = \kappa$ for $\omega \leq 0$ and $\beta = -\kappa$ for $\omega > 0$. ω is the winding number of $\Lambda(t) \neq 0$ tracing through the unit circle. For linear systems

$$\mathbf{u}_x + \mathbf{B}\mathbf{u}_y + \mathbf{C}\mathbf{u} = \mathbf{f} \text{ in } D$$

we consider I.N. Vekua's generalized analytic functions, and if the spectrum of $\mathbf{B}$ is smooth enough, for boundary conditions $\mathbf{r}\mathbf{u} = \boldsymbol{\psi}$ on the unit circle, the corresponding Fredholm indices will be obtained.