

Extensions of abstract Loewner chains and spirallikeness

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Abstract

We generate a variety of Loewner chains on the Euclidean unit ball in \mathbb{C}^n by extending chains from lower-dimensional disks or balls. Using these extended Loewner chains, we produce an assortment of spirallike mappings. Because of the Loewner chains used, these spirallike mappings are extensions, via either a modified Roper–Suffridge extension operator introduced by the author or a perturbation of the Pfaltzgraff–Suffridge extension operator, of lower-dimensional spirallike mappings. The Loewner chains under consideration are not normalized, but are of order p , meaning only a locally uniform local L^p -continuity condition is imposed on the real parameter of the family. Therefore the resulting spirallike mappings are not normalized and may be spirallike with respect to a boundary point. Furthermore, mappings are produced that satisfy a generalized form of spirallikeness with respect to a locally integrable operator-valued function A on $[0, \infty)$ rather than a fixed linear operator. There is a natural link between the function $\|A(\cdot)\|$ being locally L^p and the L^p -continuity condition on the corresponding Loewner chain. Despite the abstract nature of these results, they remain novel even in the case where A is constant and the mappings are normalized; that is, we obtain new normalized biholomorphic mappings that are spirallike with respect to a linear operator.