Workshop dedicated to the memory of Professor Gabriela Kohr
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

Cluj-Napoca, Romania 1 – 3 December 2021

Jack Lemma, starlikeness and k-symmetry in $\mathbb{C}^n$

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Abstract

Let $\mathbb{B}^n$ and $\langle \cdot, \cdot \rangle$ be the open unit ball and the Euclidean inner product in $\mathbb{C}^n$, respectively. Many authors considered the family St of biholomorphic mappings $f : \mathbb{B}^n \to \mathbb{C}^n, f(0) = 0, Df(0) = I$, with starlike domain $f(\mathbb{B}^n)$. Suffridge [Su] proved that a locally biholomorphic normalized map $f : \mathbb{B}^n \to \mathbb{C}^n$ belongs to St, iff Re $\langle [Df(z)]^{-1} f(z), z \rangle > 0, z \in \mathbb{B}^n \setminus \{0\}$. The subject of the lecture is a similar sufficient condition for a family $S(k), k \geq 2$, of locally biholomorphic maps. To define the $S(k)$ we use a unique partition [LP] $f = \sum_{j=0}^{k-1} f_{j,k}$ with components $f_{j,k}$ such that $f_{j,k}(\varepsilon z) = \varepsilon^j f_{j,k}(z), z \in \mathbb{B}^n$, where ε is the generator $\exp(\frac{2\pi i}{k})$ of the cyclic group of k^{th} roots of unity. Let $S(k), k \geq 2$, be a family of locally biholomorphic and normalized mappings $f : \mathbb{B}^n \to \mathbb{C}^n$, such that

$$\text{Re } \left\langle [Df(z)]^{-1} f_{1k}(z), z \right\rangle > 0, z \in \mathbb{B}^n \setminus \{0\}.$$

A motivation for the family $S(k)$ was a problem from [Lic3] and solved in [HK]. Some properties of $S(k)$ are given in [Lic3]. The idea of the proof of main result cames from papers [Lic2], [KL] and bases on a $\mathbb{C}^n$-version of Jack Lemma [Lic1].

Joint work with Renata Długosz.

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