

Logarithmic coefficients for some subclasses of univalent functions

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Abstract

If \mathcal{S} denotes the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic and univalent in the open unit disk \mathbb{U} , then the logarithmic coefficients $\gamma_n(f)$ of the function $f \in \mathcal{S}$ are defined by $\log \frac{f(z)}{z} = 2 \sum_{n=1}^{\infty} \gamma_n(f) z^n$.

Assuming that the function φ is univalent in the unit disk \mathbb{U} , and satisfies $\varphi(0) = 1$, and using the principle of the subordination, Ma and Minda [4] introduced the classes

$$\mathcal{S}^*(\varphi) := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi(z) \right\}, \text{ and } \mathcal{C}(\varphi) := \left\{ f \in \mathcal{A} : 1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z) \right\}.$$

Some special subclasses of the class $\mathcal{S}^*(\varphi)$ and $\mathcal{C}(\varphi)$ play a significant role in the *Geometric Function Theory* of one variable functions motivated by their geometric properties.

Based on some recent joint works, we will show some recent results connected with coefficient bounds of the above mentioned subclasses. The proofs of the main results are based on some fruitful method of Prokhorov and Szynal, Briot-Bouquet differential subordinations, etc.

We determined bounds of the difference of the moduli of successive coefficients, that is $||a_{n+1}| - |a_n||$ for the classes $\mathcal{S}^*(\varphi)$, and $\mathcal{C}(\varphi)$. For various special functions φ corresponding consequences of the main results are also presented, that incorporate some known results as the special cases.

Also, a few conjectures and some partial solutions are given.

Joint work with Ebrahim Analouei Adegani.

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2. D. Alimohammadi, E. A. Adegani, T. Bulboacă, N. E. Cho, *Logarithmic coefficients bounds and coefficient conjectures for classes associated with convex functions*, J. Funct. Spaces, vol. 2021, Article ID 6690027, 7 pages, 2021.
3. E. A. Adegani, T. Bulboacă, N. Hameed Mohammed, P. Zaprawa, *Solution of logarithmic coefficients conjectures for some classes of convex functions*, /submitted/.
4. W. C. Ma, D. Minda, *A unified treatment of some special classes of univalent functions*, In Proceedings of the Conference on Complex Analysis (Tianjin, 1992); Internat. Press, Cambridge, MA, USA, 1992, 157-169.