Babeş-Bolyai University of Cluj-Napoca
Faculty of Mathematics and Computer Science

Workshop dedicated to the memory of
Professor Gabriela Kohr

Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

Book of Abstracts

1 – 3 December 2021
Cluj-Napoca, Romania

Workshop dedicated to the memory of Professor Gabriela Kohr
Geometric Function Theory in Several Complex Variables and Complex Banach Spaces

Cluj-Napoca, Romania   1 – 3 December 2021

Invited Speakers

- **Lucian Beznea** - Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

- **Filippo Bracci** - Tor Vergata University, Rome, Italy

- **Teodor Bulboacă** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Cho-Ho Chu** - Queen Mary, University of London, UK

- **Mihai Cristea** - University of Bucharest, Romania

- **Paula Curt** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Tamás Darvas** - University of Maryland, USA

- **Mark Elin** - ORT Braude College, Karmiel, Israel

- **Anatoly Golberg** - Holon Institute of Technology, Holon, Israel

- **Ian Graham** - University of Toronto, Toronto, Canada

- **Hidetaka Hamada** - Kyushu Sangyo University, Fukuoka, Japan

- **Ikkei Hotta** - Yamaguchi University, Ube, Japan

- **Mihai Iancu** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Cezar Joiţa** - Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

- **Stanislawa Kanas** - University of Rzeszow, Poland

- **Massimo Lanza de Cristoforis** - University of Padova, Italy

- **Laszlo Lempert** - Purdue University, West Lafayette, USA

- **Piotr Liczberski** - Lodz University of Technology, Poland

- **Sergey E. Mikhailov** - Brunel University London, UK

- **Jerry R. Muir Jr.** - University of Scranton, Scranton, PA, USA

- **Victor Nistor** - Université de Lorraine, Metz, France

- **Mihai Pascu** - Transilvania University of Braşov, Romania

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- **Adrian Petruşel** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Cornel Pintea** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Radu Precup** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **Oliver Roth** - University of Wuerzburg, Germany

- **Grigore Şt. Sălăgean** - Babeş-Bolyai University, Cluj-Napoca, Romania

- **David Shoikhet** - HIT & Braude College, Israel

- **Toshiyuki Sugawa** - Tohoku University, Sendai, Japan

- **Matti Vuorinen** - University of Turku, Finland

- **Wolfgang L. Wendland** - University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH, Germany

Main organizer

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Connections between the Dirichlet and the Neumann problem

Lucian Beznea

Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

We provide an explicit solution of the generalized solution of the Neumann problem for the Laplace operator, based on a representation of the solution on the unit ball in $\mathbb{R}^n$, in terms of the solution of an associated Dirichlet problem. The representation is extended to other operators besides the Laplacian, to smooth simply connected planar domains, and to the infinite-dimensional Laplacian on the unit ball of an abstract Wiener space, providing in particular an explicit solution for the Neumann problem in this case. The talk is based on joint works with Mihai N. Pascu (Braşov, Romania) and Nicoale R. Pascu (Kennesaw State University, USA).

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Continuous extension of bihomolorphisms in higher dimension

Filippo Bracci

Tor Vergata University, Rome, Italy

Abstract

In dimension one, the Carathéodory prime end theory gives a precise characterization of those Riemann maps which extends continuously (or have non-tangential limits) in terms of the boundary of the image domains. In higher dimension Carathéodory prime end theory is available for quasi-conformal mappings—a subject which received a great contribution from Gabriela Kohr. However, in general biholomorphisms are not quasi-conformal in dimension greater than one. The renowned Fefferman's extension theorem shows that biholomorphisms among bounded smooth strongly pseudoconvex domains extends C^∞ up to the boundary. Such result has been "localized" and extended using mainly CR geometry. Here I present a different strategy for studying continuous extension of biholomorphisms based on variation of Gromov's hyperbolicity theory and "visibility". In particular, with these techniques one can prove the so-called Muir-Suffridge conjecture: every convex map from the ball extends as a homeomorphism up to the boundary (where, in case of unbounded domains one has to consider the so-called end compactification). This can be generalized to wider classes of domains.

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Logarithmic coefficients for some subclasses of univalent functions

Teodor Bulboacă

Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

If $\mathcal{S}$ denotes the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic and univalent in the open unit disk $\mathbb{U}$, then the logarithmic coefficients $\gamma_n(f)$ of the function $f \in \mathcal{S}$ are defined by $\log \dfrac{f(z)}{z} = 2 \sum_{n=1}^{\infty} \gamma_n(f) z^n$.

Assuming that the function φ is univalent in the unit disk $\mathbb{U}$, and satisfies $\varphi(0) = 1$, and using the principle of the subordination, Ma and Minda [4] introduced the classes

$$\mathcal{S}^*(\varphi) := \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi(z) \right\}, \text{ and } \mathcal{C}(\varphi) := \left\{ f \in \mathcal{A} : 1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z) \right\}.$$

Some special subclasses of the class $\mathcal{S}^*(\varphi)$ and $\mathcal{C}(\varphi)$ play a significant role in the *Geometric Function Theory* of one variable functions motivated by their geometric properties.

Based on some recent joint works, we will show some recent results connected with coefficient bounds of the above mentioned subclasses. The proofs of the main results are based on some fruitful method of Prokhorov and Szynal, Briot-Bouquet differential subordinations, etc.

We determined bounds of the difference of the moduli of successive coefficients, that is $\left| |a_{n+1}| - |a_n| \right|$ for the classes $\mathcal{S}^*(\varphi)$, and $\mathcal{C}(\varphi)$. For various special functions φ corresponding consequences of the main results are also presented, that incorporate some known results as the special cases.

Also, a few conjectures and some partial solutions are given.

Joint work with Ebrahim Analouei Adegani.

1. D. Alimohammadi, E. A. Adegani, T. Bulboacă, N. E. Cho, *Logarithmic coefficients for classes related to convex functions*, Bull. Malays. Math. Sci. Soc., vol. 44, no. 4, 2021, 2659-2673.

2. D. Alimohammadi, E. A. Adegani, T. Bulboacă, N. E. Cho, *Logarithmic coefficients bounds and coefficient conjectures for classes associated with convex functions*, J. Funct. Spaces, vol. 2021, Article ID 6690027, 7 pages, 2021.

3. E. A. Adegani, T. Bulboacă, N. Hameed Mohammed, P. Zaprawa, *Solution of logarithmic coefficients conjectures for some classes of convex functions*, /submitted/.

4. W. C. Ma, D. Minda, *A unified treatment of some special classes of univalent functions*, In Proceedings of the Conference on Complex Analysis (Tianjin, 1992); Internat. Press, Cambridge, MA, USA, 1992, 157-169.

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Bounded symmetric domains and function theory

Cho-Ho Chu

Queen Mary, University of London, UK

Abstract

We discuss the algebraic structures of bounded symmetric domains and some applications to function theory on these domains.

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On some cluster sets problems

Mihai Cristea

University of Bucharest, Romania

Abstract

We extend some theorems of Iversen and Tsuji concerning cluster sets and boundary cluster sets of holomorphic plane mappings in the class of so called ring mappings. Such mappings defined by satisfying a modular inequality were intensively studied in the past 20 years by many mathematicians like Cristea, Golberg, Kovtoniuk, Martio, Ryazanov, Salimov, Sevost'yanov, Srebro, Yakubov and many geometric properties of the well-known quasiregular mappings still hold in this class of mappings.

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Some remarks on certain invariant geometric properties in Hele-Shaw flows

Paula Curt

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

We survey some recent results that occur from fluid dynamics, problems that are interesting for geometric function theory specialists. We study certain geometric properties of the moving boundary in the case of two-dimensional viscous fluid flows in Hele-Shaw cells under injection. We discuss the invariance in time of free boundary for bounded and unbounded (with bounded complement) domains. To this end special classes of univalent functions which admit an explicit geometric interpretation are considered.

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The volume of pseudoeffective line bundles

Tamás Darvas
University of Maryland, College Park, USA

Abstract

Let L be a line bundle with positive singular Hermitian metric he^{-u}, on an n-dimensional compact Kähler manifold X. Let h_k be the dimension of the space of global sections that are L^2 integrable with respect to the weight e^{-ku}. We show that the limit of h_k/k^n exists, and equals the non-pluripolar volume of the I-model potential associated to u. Joint work with Mingchen Xia.

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Geometric features of non-linear resolvents
in the unit disk

Mark Elin

ORT Braude College, Karmiel , Israel

Abstract

Let f be an infinitesimal generator of a one-parameter semigroup of holomorphic self-mappings on the open unit disk vanishing at zero. Denote by $\mathcal{J} = \{(I + rf)^{-1},\ r > 0\}$, the family of resolvents of such generators. The aim of this talk is to present properties of such resolvents as univalent functions obtained in [1–3].

We establish, in particular, covering and distortion theorems that enable us to estimate order of starlikeness and order of strong starlikeness for resolvents. It turns out that order of starlikeness grows from $\frac{1}{2}$ to 1. In turn, this implies that the family of normalized resolvents converges to the identity map. Also, any resolvent admits quasiconformal extension to the complex plane $\mathbb{C}$. We prove that any element of $\mathcal{J}$ is also a generator and obtain some characteristics of semigroups generated by them. The existence/non-exoistence of repelling fixed points of resolvents is also studied.

1. M. Elin and F. Jacobzon, Some geometric features of non-linear resolvents, 2021 available in arXiv: https://arxiv.org/pdf/2104.00758.

2. M. Elin and F. Jacobzon, Estimates on some functionals over non-linear resolvents, 2021, available in arXiv: https://arxiv.org.pdf/2105.09582

3. M. Elin, D. Shoikhet and T. Sugawa, Geometric properties of the nonlinear resolvent of holomorphic generators, *J. Math. Anal. Appl.* **483** (2020), No. 123614.

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Absolute continuity in higher dimensions

Anatoly Golberg

Holon Institute of Technology, Holon, Israel

Abstract

Geometric Function Theory lies at the core of two distinguished fields of Mathematics, namely, Geometry and Analysis, and has various fundamental applications. The main goal of the present talk is to survey the absolute continuity and some related important notions of geometric function theory of several real variables. We also provide the main relationships between various classes of mappings whose definitions rely on metric approaches and techniques: finitely bi-Lipschitz mappings, quasisymmetric mappings, quasimöbius and quasiconformal mappings, mappings of finite metric and area distortion. Several illustrated examples are also presented.

The talk is based on joint works with Elena Afanas'eva (Institute of Applied Mathematics and Mechanics of the NAS of Ukraine).

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Loewner PDE and univalent subordination chains in infinite dimensions

Hidetaka Hamada

Kyushu Sangyo University, Fukuoka, Japan

Abstract

In this talk, we give the existence and uniqueness result for the solutions of the Loewner partial differential equation (Loewner PDE) in the case of separable reflexive complex Banach spaces. Related results on univalent subordinations and the transition mappings associated with univalent subordination chains are obtained. This is a joint work with Gabriela Kohr.

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Non-commutative probability and Loewner chains

Ikkei Hotta

Yamaguchi University, Ube, Japan

Abstract

Recently, very interesting interrelation with non-commutative probability and Loewner theory was established by Franz, Hasebe and Schleissinger. In their work, a bijection between "multiplicative Loewner chains on the unit disk" and "unitary multiplicative processes of monotonically independent increments" was formulated. In this talk, as an extension of their work, we introduce another bijection between "multiplicative Loewner chains on the unit disk" and "unitary multiplicative processes of classical (tensor) independent increments". This talk is based on a joint work with Takahiro Hasebe.

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On certain polynomially convex sets in $\mathbb{C}^n$

Mihai Iancu

Babeș-Bolyai University, Cluj-Napoca, Romania

Abstract

First we discuss a criterion for two Φ-like domains in $\mathbb{C}^n$ to form a Runge pair. Next we consider geometric characterizations (starlikeness, spiralikeness and convexity) with respect to automorphisms of $\mathbb{C}^n$ for certain biholomorphic mappings $f : \mathbb{B}^n \to \mathbb{C}^n$, defined on the Euclidean unit ball, with $\overline{f(\mathbb{B}^n)}$ polynomially convex. In particular, we present an extension of a result of Arosio, Bracci and Wold on convexshapelike domains. Certain examples and applications are also discussed.

This talk is based on joint work with Hidetaka Hamada and Gabriela Kohr.

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The local image problem for complex analytic maps

Cezar Joiţa

Simion Stoilow Institute of Mathematics of the Romanian Academy, Bucharest, Romania

Abstract

For holomorphic map germs $(X, x) \to (Y, y)$ between germs of complex spaces, we discuss the problem of defining their image. We prove a criterion for such a map germ to be open, solving in this way a conjecture by A. Huckleberry. Joint work with Mihai Tibăr.

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Conformal and harmonic mappings and planar domains

Stanislawa Kanas

University of Rzeszów, Poland

Abstract

This talk focuses on the relations between conformal and harmonic mappings and connected planar domains in $\mathbb{C}$ that depend smoothly on a real parameters. We present convex and non-convex domains with its applications in Geometric Functions Theory.

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About the research activity of Professor Gabriela Kohr

Mirela Kohr

Babeș-Bolyai University, Cluj-Napoca, Romania

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Microscopic behaviour of the solutions of a transmission problem for the Helmholtz equation. A functional analytic approach

Massimo Lanza de Cristoforis
University of Padova, Italy

Abstract

We consider a transmission problem for the Helmholtz equation in a domain with a small inclusion of size $\epsilon > 0$ and we analyze the microscopic behavior of the solutions, *i.e.*, the behavior of the rescaled solutions, as $\epsilon > 0$ tends to zero by an approach that is alternative to that of asymptotic expansions.

Joint work with Tuğba Akyel, Maltepe University, Istanbul.

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Plurisubharmonic domination

Laszlo Lempert
Purdue University, West Lafayette, USA

Abstract

Plurisubharminic domination is a concept in complex analysis of infinite dimensional manifolds that can, in some cases, replace the device of compact exhaustion, much used in finite dimensional analysis. The plan of the talk is to introduce the notion of plurisubharmonic functions and pseudoconvex domains in Banach spaces, formulate the domination theorem (due to Patyi and myself), and indicate the proof and applications.

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Jack Lemma, starlikeness and k-symmetry in $\mathbb{C}^n$

Piotr Liczberski

Lodz University of Technology, Poland

Abstract

Let $\mathbb{B}^n$ and $\langle \cdot, \cdot \rangle$ be the open unit ball and the Euclidean inner product in $\mathbb{C}^n$, respectively. Many authors considered the family St of biholomorphic mappings $f : \mathbb{B}^n \to \mathbb{C}^n, f(0) = 0, Df(0) = I$, with starlike domain $f(\mathbb{B}^n)$. Suffridge [Su] proved that a locally biholomorphic normalized map $f : \mathbb{B}^n \to \mathbb{C}^n$ belongs to St, iff Re $\langle [Df(z)]^{-1}f(z), z \rangle > 0, z \in \mathbb{B}^n \setminus \{0\}$. The subject of the lecture is a similar sufficient condition for a family $S(k), k \geq 2$, of locally biholomorphic maps. To define the $S(k)$ we use a unique partition [LP] $f = \sum_{j=0}^{k-1} f_{j,k}$ with components $f_{j,k}$ such that $f_{j,k}(\varepsilon z) = \varepsilon^j f_{j,k}(z), z \in \mathbb{B}^n$, where ε is the generator $\exp(\frac{2\pi i}{k})$ of the cyclic group of kth roots of unity. Let $S(k), k \geq 2$, be a family of locally biholomorphic and normalized mappings $f : \mathbb{B}^n \to \mathbb{C}^n$, such that

$$\text{Re } \langle [Df(z)]^{-1} f_{1k}(z), z \rangle > 0, z \in \mathbb{B}^n \setminus \{0\}.$$

A motivation for the family $S(k)$ was a problem from [Lic3] and solved in [HK]. Some properties of $S(k)$ are given in [Lic3]. The idea of the proof of main result cames from papers [Lic2], [KL] and bases on a $\mathbb{C}^n$-version of Jack Lemma [Lic1].

Joint work with Renata Długosz.

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Layer and volume potentials of the Stokes system with non-smooth anisotropic viscosity tensor and some applications

Sergey E. Mikhailov

Brunel University London, UK

Abstract

The first aim of this research is to develop a volume and layer potential theory in L_2-based weighted Sobolev spaces on Lipschitz bounded and exterior domains of $\mathbb{R}^n$, $n \geq 3$, for the anisotropic Stokes system with L_∞ viscosity coefficient tensor satisfying an ellipticity condition for symmetric matrices. To do this, we explore equivalent mixed variational formulations and prove the well-posedness of some transmission problems for the anisotropic Stokes system in Lipschitz domains in weighted Sobolev spaces. These results are used to define the Newtonian and layer potentials and to obtain their properties. Then we analyze well-posedness of the exterior Dirichlet, Neumann and mixed problems for the Stokes system with L_∞ elliptic coefficient tensor satisfying a relaxed ellipticity condition. Finally, we prove the existence of a solution for a transmission problem in complementary Lipschitz domains in $\mathbb{R}^3$ for the anisotropic Navier-Stokes system with data in L_2-based weighted Sobolev spaces. The analysis relies on an existence result for a Dirichlet-transmission problem for the anisotropic Navier-Stokes system in a family of bounded domains, and on the Leray-Schauder fixed point theorem.

Presentation is based on the results obtained in collaboration with M. Kohr (Cluj-Napoca, Romania) and W.L.Wendland (Stuttgart, Germany) and published in the paper:

M. Kohr, S.E. Mikhailov, W.L. Wendland, Layer potential theory for the anisotropic Stokes system with variable L_∞ symmetrically elliptic tensor coefficient. *Math. Meth. Appl. Sci.*, **44** (2021), 9641–9674.

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Extensions of abstract Loewner chains and spirallikeness

Jerry R. Muir Jr.
University of Scranton, PA, USA

Abstract

We generate a variety of Loewner chains on the Euclidean unit ball in $\mathbb{C}^n$ by extending chains from lower-dimensional disks or balls. Using these extended Loewner chains, we produce an assortment of spirallike mappings. Because of the Loewner chains used, these spirallike mappings are extensions, via either a modified Roper–Suffridge extension operator introduced by the author or a perturbation of the Pfaltzgraff–Suffridge extension operator, of lower-dimensional spirallike mappings. The Loewner chains under consideration are not normalized, but are of order p, meaning only a locally uniform local L^p-continuity condition is imposed on the real parameter of the family. Therefore the resulting spirallike mappings are not normalized and may be spirallike with respect to a boundary point. Furthermore, mappings are produced that satisfy a generalized form of spirallikeness with respect to a locally integrable operator-valued function A on $[0,\infty)$ rather than a fixed linear operator. There is a natural link between the function $\|A(\cdot)\|$ being locally L^p and the L^p-continuity condition on the corresponding Loewner chain. Despite the abstract nature of these results, they remain novel even in the case where A is constant and the mappings are normalized; that is, we obtain new normalized biholomorphic mappings that are spirallike with respect to a linear operator.

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Lie algebras of vector fields and analysis of boundary value problems

Victor Nistor
Université de Lorraine, Metz, France

Abstract

An approach, due to Epstein, Melrose, and Mendoza to the analysis on strictly pseudoconvex domains, is based on certain Lie algebra of vector fields on the domain. This approach is useful in other settings and can indeed be generalized. I will begin by reviewing some classical results on the analysis on compact manifolds and on manifolds with conical points. It turns out that many of these classical results generalize to a larger class of singular or non-compact spaces with nice ends obtained by using suitable classes of vector fields. These vector fields model the geometry at infinity. One obtains, for instance, the well-posedness of the Dirichlet problem on general n-dimensional polyhedral domains. Another typical result is to obtain Fredholm conditions for the natural operators on singular domains and spaces.

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Connections between the Dirichlet, Neumann, and the mixed Dirichlet-Neumann problems for the Laplacian

Mihai Pascu

Transilvania University of Braşov, Romania

Abstract

The Neumann problem for a certain partial differential equation is in general harder to solve than the corresponding Dirichlet problem. In [1], [2], [3], we have shown that under certain smoothness assumptions on the boundary of the domain and of the boundary data, the Dirichlet and the Neumann problems (for the Laplace's equation and some other related PDE) are in fact "equally harder" to solve, in the sense that solving one problem leads immediately to the solution of the other (in particular, this gave an explicit representation of the Dirichlet-to-Neumann operator).

In the present talk we extend these results, by establishing a connection between the mixed Dirichlet-Neumann and the Dirichlet problem for the Laplace operator, and by exploring the case of weaker assumptions on the boundary data.

The talk is based on joint work with L. Beznea (IMAR, Bucharest, Romania) and N. R. Pascu (Kennesaw State University, Marietta, USA).

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Gabriela Kohr - a distinguished professor of the Babeş-Bolyai University

Adrian Petruşel

Babeș-Bolyai University, Cluj-Napoca, Romania

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Cluj-Napoca, Romania
1 – 3 December 2021

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Global injectivity criteria and the valence of some functions

Cornel Pintea

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

We point out the role of monotonicity in proving some global injectivity criteria. We also investigate the valence of functions subject to the hypotheses of such global injectivity criteria, except the convexity hypothesis of the domain, which is removed. For such functions the convexity number of the domain is a upper bound for their valence.

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Nonlinear perturbations of the time-dependent Schrodinger equation

Radu Precup

Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

The time-dependent Schrödinger equation is a cornerstone of quantum physics. It is representative for the role of complex analysis in studying differential systems modelling wave processes with phase differences. The talk will focus on some classes of nonlinear state-dependent forces capable of maintaining wave movements with an initial generation. Our analysis is based on the theory of nonlinear operators.

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Critical points, Blaschke products and beyond

Oliver Roth
University of Wuerzburg, Germany

Abstract

We survey some recent results concerning boundary rigidity properties of holomorphic self-maps of the unit disk, critical sets of Hardy space functions, zero sets of Bergman space functions, and the invariant subspace problem for Bergman spaces (partly based on joint work with Filippo Bracci and Daniela Kraus).

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Gabriela Kohr – Mathematician, Professor, Colleague

Grigore Şt. Sălăgean
Babeş-Bolyai University, Cluj-Napoca, Romania

Abstract

About Gabriela Kohr's life and professional activity. Points of view as her former professor, head of the department and colleague.

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Rigidity of holomorphic mappings and semigroups

David Shoikhet

HIT & Braude College, Israel

Abstract

There is a long history associated with the problem of iterating nonexpansive and holomorphic mappings and finnding their fixed points. Historically, complex dynamics and geometrical function theory have been intensively developed from the beginning of the twentieth century. They provide the foundations for broad areas of mathematics. In the last fifty years the theory of holomorphic mappings on complex spaces has been studied by many mathematicians with many applications to nonlinear analysis, functional analysis, differential equations, classical and quantum mechanics. The laws of dynamics are usually presented as equations of motion which are written in the abstract form of a dynamical system: $\frac{dx}{dt} + f(x) = 0$, where x is a variable describing the state of the system under study, and f is a vector-function of x. The study of such systems when f is a monotone or an accretive (generally non-linear) operator on the underlying space has recently been the subject of much research by analysts working on quite a variety of interesting topics, including boundary value problems, integral equations and evolution problems. In this talk we give a brief description of the classical statements with their modern interpretations for discrete and continuous semigroups of hyperbolically nonexpansive mappings in Hilbert and Banach spaces. We also present some special recent achivments for the one-dimensional case. This talk mostly based on joint works with M. Elin and T. Sugawa.

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Nonlinear resolvents for semigroups on the upper half-plane

Toshiyuki Sugawa

Tohoku University, Sendai, Japan

Abstract

Nonlinear resolvents are known to exist for infinitesimal generators of semigroups of holomorphic self-maps of a bounded convex domain (even in higher dimensions). However, the case of unbounded domains is less known. In this talk, we examine the case when the domain is the upper half-plane. Indeed, in this case, there are infinitesimal generators which do not admit nonlinear resolvents for all the time $t \geq 0$. On the other hand, for a generic choice of infinitesimal generators, the nonlinear resolvents exist for $t \geq 0$. The present talk is based on joint work with Ikkei Hotta and Sebastian Schleissinger.

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Intrinsic metrics in geometric function theory

Matti Vuorinen
University of Turku, Finland

Abstract

Two generalizations of the hyperbolic metric are discussed: the triangle ratio metric and the visual angle metric. We also report experimental results concerning conformal capacities of planar condensers and bounds for their capacities in terms of the hyperbolic perimeter. This talk is based on the following joint works:

- P. Hariri, R. Kl'en, and M. Vuorinen: Conformally Invariant Metrics and Quasiconformal Mappings.- Springer 2020.

- G. Wang, M. Vuorinen, and X. Zhang: On cyclic quadrilaterals in Euclidean and hyperbolic geometries.- Publ. Math. Debrecen 99/1-2 (2021), 123–140.

- M. M.S. Nasser, O. Rainio, and M. Vuorinen: Condenser capacity and hyperbolic perimeter+MATLAB code.- Comp. Math. Appl. (to appear).

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First order elliptic systems in the plane

Wolfgang L. Wendland

University of Stuttgart, Institute for Applied Analysis and Numerical Simulation & SIMTECH,
Germany

Abstract

The lecture is based on [Elliptic Systems in the Plane, Pitman, London 1979] and is concerned with first order real systems of n linear equations for vector-valued functions $\mathbf{u} \subset \mathbb{R}^n$, elliptic in the sense of Petrovski. In the simplest case and $n = 2$, these are the Cauchy–Riemann equations for holomorphic functions w in $D \subset \mathbb{C}$. For Riemann–Hilbert boundary value problems in the unit disc D with boundary conditions $Re(\Lambda w)|_{|z|=1} = \varphi$, for w in the Sobolev space $W_2^\varrho(D)$ Fritz Noether's theorem is valid and the Fredholm index is $\kappa = 1 - 2\omega = \alpha - \beta$ where $\alpha = \kappa$ for $\omega \leq 0$ and $\beta = -\kappa$ for $\omega > 0$. ω is the winding number of $\Lambda(t) \neq 0$ tracing through the unit circle. For linear systems

$$\mathbf{u}_x + \mathbf{B}\mathbf{u}_y + \mathbf{C}\mathbf{u} = \mathbf{f} \text{ in } D$$

we consider I.N. Vekua's generalized analytic functions, and if the spectrum of $\mathbf{B}$ is smooth enough, for boundary conditions $\mathbf{r}\mathbf{u} = \boldsymbol{\psi}$ on the unit circle, the corresponding Fredholm indices will be obtained.