ENTIRE BOUNDED SOLUTIONS VERSUS FIXED POINTS FOR NONLINEAR ELLIPTIC EQUATIONS WITH INDEFINITE WEIGHT

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Abstract. We establish a necessary and sufficient condition for the existence of an entire distributional solution for a general class of nonlinear elliptic equations with variable potential and nondecreasing nonlinear term. Our result establishes the relationship between the Green function and the growths of the weight and of the nonlinear term. The main result also points out the connection with a fixed point problem for an integral operator.
Key Words and Phrases: nonlinear elliptic equation, entire solution, fixed point, Green function.
2010 Mathematics Subject Classification: 35B50, 35J61, 35J67, 35J75, 58J32, 47H10.

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