Applications - classical probability distributions

October 26, 2010

1 Discrete Distributions

1.1 Binomial distribution

1. Plot pdf and cdf of a binomial distribution with $p = 0.2$ and $n = 10$.

2. A Quality Assurance inspector tests 200 circuit boards a day. If 2% of the boards have defects, what is the probability that the inspector will find no defective boards on any given day? What is the most likely number of defective boards the inspector will find?

3. Suppose that a lot of 300 electrical fuses contains 5% defectives. If a sample of five fuses is tested, find the probability of observing at least one defective.

4. Experience has shown that 30%of all persons afflicted by a certain illness recover. A drug company has developed a new medication. Ten people with the illness were selected at random and injected with the medication; nine recovered shortly thereafter. Suppose that the medication was absolutely worthless. What is the probability that at least nine of ten injected with the medication will recover?

1.2 Geometric distribution

1. Plot a pdf and a cdf of a geometric distribution with $p = 0.5$.

2. Suppose you toss a fair coin repeatedly. If the coin lands face up (heads), that is a success. What is the probability of observing exactly three tails before getting a heads? What is the probability of observing three or fewer tails before getting a heads?

3. 40% of the assembled ink-jet printers are rejected at the inspection station. Find the probability that the first acceptable ink-jet printer is the thirs one inspected.

4. Prove the memoryless property for geometric distribution.

1.3 Negative binomial distribution

1. Plot the pdf and the cdf of a negative binomial distribution for $r = 3$ and $p = 0.5$
2. A geological study indicates that an exploratory oil well drilled in a particular region should strike oil with the probability 0.2. Find the probability that the oil strike comes on the fifth well drilled and on the first five wells drilled.
3. 40% of the assembled ink-jet printers are rejected at the inspection station. Find the probability that the first printer inspected is the second acceptable printer.

1.4 Poisson Distribution

1. Plot the pdf and the cdf of a Poisson distribution for $\lambda = 5$.
2. Suppose that a random system of police patrol is devised so that a patrol officer may visit a given beat location $Y = 0, 1, 2, \ldots$ times per half-hour period, with each location being visited an average of once per time period. Assume that Y possesses, approximately, a Poisson probability distribution. Calculate the probability that the patrol officer will miss a given location during a half-hour period. What is the probability that it will be visited once? Twice? At least once?
3. A computer hard disk manufacturer has observed that flaws occur randomly in the manufacturing process at the average rate of two flaws in a 4 Gb hard disk and has found this rate to be acceptable. What is the probability that a disk will be manufactured with no defects?
4. Consider a Quality Assurance department that performs random tests of individual hard disks. Their policy is to shut down the manufacturing process if an inspector finds more than four bad sectors on a disk. What is the probability of shutting down the process if the mean number of bad sectors (λ) is two?
5. A repair person is "beeped" each time there is a call service. The number of beeps is Poisson with $\lambda = 2$ per hour. Find the probability of three beeps in the next hour, and the probability of more than three beeps i an 1-hour period.
6. The lead time demand in an inventory system is the accumulation of demand for an item at which an order is placed until the order is received

$$L = \sum_{i=1}^{T} D_i,$$

where L is the lead-time demand, D_i is the demand during the ith time period, and T is the number of time periods during the lead time. Both D_i and T may be random variables. An inventory manager desires that the probability of a stockout not exceed a certain fraction (e.g. 5%) during the lead time. The roerder point is the level of inventory at which a new order is placed. Assume that the lead-time demand is poisson distributed with a mean of $\lambda = 10$ units and that a 95% protection of a stockout is desired. That is, find the smallest x such that the probability that the lead-time demand does not exceed x is $\geq 95\%$.

1.5 Hypergeometric distribution

1. Plot the pdf and the cdf of an experiment taking 20 samples from a group of 1000 where there are 50 items of the desired type.
2. Suppose you have a lot of 100 floppy disks and you know that 20 of them are defective. What is the probability of drawing 0 through 5 defective floppy disks if you select 10 at random? What is the probability of drawing zero to two defective floppies if you select 10 at random?
3. Suppose you are the Quality Assurance manager for a hard disk manufacturer. The production line turns out disks in batches of 1,000. You want to sample 50 disks from each batch to see if they have defects. You want to accept 99% there are no more than 10 defective disks in the batch. What is the maximum number of defective disks should you allow in your sample of 50?

2 Continuous Distributions

2.1 Uniform distribution

1. Plot the graphs of pdf and cdf of a $U[0, 1]$ distribution.
2. What is the probability that an observation from a uniform distribution with $a = -1$ and $b = 1$ will be less than 0.75?
3. What is the 99th percentile of the uniform distribution between -1 and 1?
4. A bus arrives every 20 minutes at a specified stop beginning at 6:40 A.M. and continuing until 8:40 A.M. A certain passenger does not know the schedule, but arrives randomly (uniformly distributed) between 7:00 A.M. and 7:30 A.M. every morning. What is the probability that the passenger waits more than 5 minutes for a bus.

2.2 Normal distribution

1. Plot the pdf and the cdf of the standard normal distribution

2. Plot on the same graph the pdf of $N(0, 1^2)$, $N(0, 2^2)$, $N(0, 3^2)$. Do the same for the cdf.

3. Check the 3σ rule for a given normal distribution.

4. Find an interval that contains 95% of the values from a standard normal distribution. Note this interval is not the only such interval, but it is the shortest. Find another, longer.

5. Lead-time demand X for an item is approximated by a $N(25, 9)$ distribution. It is desired to compute the value for led time that will be exceeded only 5% of the time, that is find x_0 such that $P(X > x_0) = 0.05$.

6. The time to pass through a queue to begin self-service at a cafeteria has been found to be $N(15, 9)$. Compute the probability that an arriving customer waits between 14 and 17 minutes.

2.3 Lognormal distribution

1. Suppose the income of a family of four in the United States follows a lognormal distribution with $\mu = ln(20,000\$)$ and $\sigma^2 = 1$. Plot the income density. What is the probability that the income be larger than 60000$.

2. The rate of return on a volatile investment is modeled as having a lognormal distribution with mean 20% and standard deviation 5%. Compute the parameters for the lognormal distribution.

2.4 Beta distribution

1. A gasoline wholesale distributor has bulk storage tanks that hold fixed supplies and are filled every Monday. Of interest to the wholesaler is the proportion of this supply that is sold during the week. Over many weeks of observation, the distributor that this proportion could be modelled by a beta distribution with $\alpha = 4$ and $\beta = 2$. Find the probability that the wholesaler will sent at least 90% of her stock in a given week.

2. Write a MATLAB script that computes the quartiles of a beta distribution, given the parameters a and b.

2.5 Triangular distribution

1. A central processor unit requirements, for programs that will execute, have a triangular distribution with $a = 0.05$ ms, $b = 1.1$ ms, and $c = 6.5$ ms. Find the probability that the CPU requirements for a random program is 2.5 ms or less.

2. Implement MATLAB functions for the pdf, cdf, icdf of a triangular distribution.

3. An electronic sensor evaluates the quality of memory chips, rejecting those thai fail. Upon demand the sensor will give the minimum and maximum number of rejects over the past 24 hours. The mean is also given. Without further information, the quality control department has assumed thet the number of rejected chips can be approximated by a triangular distribution. The current dump data indicates that the minimum number of rejected chips during any hour was zero, the maximum was 10, and the mean was 4. Find a, b, and c and the median.

4. Find conditions on a, b, c such that mean, mode and median be equal.

5. Find the variance and the median of a $Triang(a, b, c)$ r.v.

2.6 Exponential distribution

1. The median of the exponential distribution is $\frac{\ln 2}{\lambda}$. Prove this fact this fact.

2. What is the probability that an exponential random variable will be less than or equal to $1/E(X)$?

3. Let the lifetime of light bulbs be exponentially distributed with $\beta = 700$ hours. What is the median lifetime of a bulb?

2.7 Weibull distribution

1. Reproduce Figure from lecture slides.. Plot cdfs for the same distributions.

2. Find the formula for the median of a Weibull distribution.

3. The time to failure for a component screen is known to have a Weibull distribution with $\nu = 0$, $\beta = 1/3$, and $\alpha = 200$ hours. Find the mean, the variance and the probability that a unit fails before 2000 hours.

4. The time it takes for an aircraft to land and clear the runaway at a major international airport has a Weibull distribution with $\nu = 1.34$ minutes, $\beta = 0.5$ and $\alpha = 0.04$ minutes. Find the probability that an incoming airpalne will take more than 1.5 minutes to land and cler the runaway.

2.8 Gamma distribution

1. Plot gamma pdfs and cdfs for $a = 1, 2, 4$ and $\lambda = 1$.

2. Four-week summer rainfall totals in a section of the midwest United States have approximately a gamma distribution with $a = 1.6$ and $\theta = 2.0$. Find the probability to have an amount of rainfall between 3 and 57. What is the median of the rainfall?

3. Annual incomes for heads of household in a section of a city have approximately a gamma distribution with $a = 1000$ and $\theta = 20$. Find the mean and the variance of these incomes. Would you expect to find many incomes in excess of \$40,000 in this section of the city?