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COEFFICIENT INEQUALITIES FOR CERTAIN CLASSES OF ANALYTIC FUNCTIONS USING q-DERIVATIVES

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Abstract. We introduce and we study the classes $\mathcal{ST}_q\,(g,\lambda,\gamma,\alpha,\beta)$ and $\mathcal{KV}_q\,(g,\lambda,\gamma,\alpha,\beta)$ of analytic functions which are defined by making use of the q-derivative operator. Coefficient inequalities for functions in these classes are discussed. Some interesting consequences of the results are also pointed out.

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1. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which are analytic in the open disc $\mathcal{U} = \{\ z : z \in \mathbb{C} : |z| < 1\}$.

Let $f \in \mathcal{A}$ be given by (1)and g be given by

$$g(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{2}$$

The convolution or Hadamard product of $f(z)$ and $g(z)$ is denoted by $(f*g)$ and is defined as

$$(f*g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n. \tag{3}$$

An analytic function f is said to be subordinate to an analytic function g (written as $f \prec g$) if and only if there exists an analytic function ω with $\omega(0) = 0$ and $|\omega(z)| < 1$ for $z \in U$, such that $f(z) = g(\omega(z))$ for $z \in U$.

In particular, if g is univalent in U,we have the following equivalence

$$f \prec g \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

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A function f in $\mathcal{A}$ is said to be uniformly convex in $\mathcal{U}$ if f is a univalent convex function along with the property that, for every circular arc γ contained in $\mathcal{U}$, with center ζ also in $\mathcal{U}$, the image curve $f(\gamma)$ is a convex arc. The class of uniformly convex functions is denoted by UCV (see [11]). It is well known [17] that UCV if and only if $\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right) > \left|\frac{zf''(z)}{f'(z)}\right|$, $z \in \mathcal{U}$, and the corresponding class UST is defined by the relation that $f \in UST$ if and only if $\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \left|\frac{zf'(z)}{f(z)}-1\right|$, $z \in \mathcal{U}$.

Uniformly starlike and convex functions were first introduced by Goodman [12] and then studied by various other authors.

Also, a function $f \in \mathcal{A}$ is said to be in the class of uniformly convex functions of order α and type β denoted by $UC(\alpha,\beta)$ (see [2] and [6]) if

$$\operatorname{Re}\left(1+\frac{zf''(z)}{f'(z)}\right) > \alpha\left|\frac{zf''(z)}{f'(z)}\right| + \beta, \quad (\alpha \geq 0,\, 0 \leq \beta < 1;\, z \in \mathcal{U}) \tag{4}$$

and is said to be in a corresponding class denoted by $SP(\alpha,\beta)$ if

$$\operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right) > \alpha\left|\frac{zf'(z)}{f(z)}-1\right| + \beta, \quad (\alpha \geq 0,\, 0 \leq \beta < 1;\, z \in \mathcal{U}). \tag{5}$$

We note that $f(z) \in UC(\alpha,\beta) \Leftrightarrow zf'(z) \in SP(\alpha,\beta)$.

Now, we refer to a notion of q-operators i.e. q-difference operator and q-integral operator that play vital role in the theory of hypergeometric series, quantum physics and in the operator theory. The application of q-calculus was initiated by Jackson [14, 15]. He was the first mathematician who developed q-derivative and q-integral in a systematic way. Purohit and Raina [23], Kanas and Răducanu [16] have used the fractional q-calculus operators in investigations of certain classes of functions which are analytic in the open disk. A comprehensive study on applications of q-calculus in operator theory may be found in [5]. Both operators play crucial role in the theory of relativity, usually encompasses two theories by Einstein, one in special relativity and the other in general relativity. Special relativity applies to the elementary particles and their interactions, whereas general relativity applies to the cosmological and astrophysical realm, including astronomy. Special relativity theory rapidly became a significant and necessary tool for theorists and experimentalists in the new fields of atomic physics, nuclear physics and quantum mechanics.

The q-difference operator denoted as $D_q f(z)$ is defined by

$$D_q f(z) = \begin{cases} \frac{f(z)-f(qz)}{z(1-q)} & \text{for } z \neq 0, \\ f'(0) & \text{for } z = 0, \end{cases} \tag{6}$$

and $D_q^2 f(z) = D_q\left(D_q f(z)\right)$.

From (6), we have $D_q f(z) = 1 + \sum_{n=2}^{\infty} [n]_q\, a_n z^{n-1}$, where

$$[n]_q = \frac{1-q^n}{1-q}. \tag{7}$$

As $q \to 1^-, [n]_q \to n$. For a function $h(z) = z^n$, we observe that

$$D_q(h(z)) = D_q(z^n) = \frac{1-q^n}{1-q} z^{n-1} = [n]_q \, z^{n-1},$$

$$\lim_{q\to 1} D_q(h(z)) = \lim_{q\to 1} \left([n]_q \, z^{n-1} \right) = n z^{n-1} = h'(z),$$

where h' is the ordinary derivative.

As a right inverse, Jackson [14] introduced the q-integral

$$\int_0^z h(t) \mathrm{d}_q t = z(1-q) \sum_{n=0}^{\infty} q^n f\left(zq^n\right),$$

provided that the series converges. For a function $h(z) = z^n$, we observe that

$$\int_0^z h(t) \mathrm{d}_q t = \lim_{q \to 1^-} \frac{z^{n+1}}{[n+1]_q} = \frac{z^{n+1}}{n+1} = \int_0^z h(t) \mathrm{d}t,$$

where $\int_0^z h(t)\mathrm{d}t$ is the ordinary integral.

Motivated by the work of Muhammad Arif et al.[19], using q-derivative operator, we define the following subclasses:

$$\mathcal{ST}_q\left(g, \lambda, \gamma, \alpha, \beta\right) \quad \text{and} \quad \mathcal{KV}_q\left(g, \lambda, \gamma, \alpha, \beta\right)$$

of analytic function.

Definition 1.1. An analytic function $f(z)$ of the form (1) belongs to the class $\mathcal{ST}_q\left(g, \lambda, \gamma, \alpha, \beta\right)$, if and only if

$$\begin{aligned} \operatorname{Re}\left\{ \mathrm{e}^{\mathrm{i}\lambda} \left(1 - \frac{2}{\gamma} + \frac{2}{\gamma} \left(\frac{z D_q(f*g)(z)}{(f*g)(z)} \right) \right) \right\} \\ > \alpha \left| \frac{2}{\gamma} \left(\frac{z D_q(f*g)(z)}{(f*g)(z)} - 1 \right) \right| + \beta \cos\lambda \end{aligned} \tag{8}$$

where $\alpha \geq 0, \gamma \in \mathbb{C} \setminus \{0\}$, λ is a real with $|\lambda| < \frac{\pi}{2}$, $0 < q < 1$ and $0 \leq \beta < 1$.

Definition 1.2. An analytic function $f(z)$ of the form (1) belongs to the class $\mathcal{KV}_q\left(g, \lambda, \gamma, \alpha, \beta\right)$, if and only if

$$\begin{aligned} \operatorname{Re}\left\{ \mathrm{e}^{\mathrm{i}\lambda} \left(1 - \frac{2}{\gamma} + \frac{2}{\gamma} \left(\frac{D_q(z D_q(f*g)(z))}{D_q(f*g)(z)} \right) \right) \right\} \\ > \alpha \left| \frac{2}{\gamma} \left(\frac{D_q(z D_q(f*g)(z))}{D_q(f*g)(z)} - 1 \right) \right| + \beta \cos\lambda \end{aligned} \tag{9}$$

where $\alpha \geq 0, \gamma \in \mathbb{C} \setminus \{0\}$, λ is a real with $|\lambda| < \left(\frac{\pi}{2}\right)$, $0 < q < 1$ and $0 \leq \beta < 1$.

2. MAIN RESULTS

THEOREM 2.1. *Let* $f(z) \in \mathcal{ST}_q(g, \lambda, \gamma, \alpha, \beta)$ *with* $0 \le \alpha \le \beta$. *Then* $f(z) \in \mathcal{ST}_q\left(g, \lambda, \gamma, 0, \frac{\beta-\alpha}{1-\alpha}\right)$.

Proof. Let $f(z) \in \mathcal{ST}_q(g, \lambda, \gamma, \alpha, \beta)$. Then we obtain

(10)
$$\begin{aligned}\operatorname{Re} e^{i\lambda}\left\{1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)\right\} &> \alpha\left|\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}-1\right)\right|+\beta\cos\lambda \\ &> \alpha \operatorname{Re} e^{i\lambda}\left(1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)\right) \\ &\quad -\alpha \operatorname{Re}\, e^{i\lambda}+\beta\cos\lambda\end{aligned}$$

and this implies

$$\operatorname{Re}\left\{e^{i\lambda}\left(1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)\right)\right\} > \frac{\beta-\alpha}{1-\alpha}\cos\lambda. \tag{11}$$

Also if $0 \le \alpha \le \beta$, the we can easily obtain $0 \le \frac{\beta-\alpha}{1-\alpha} < 1$, and this completes the proof. □

THEOREM 2.2. *If* $f(z) \in \mathcal{ST}_q(g, \lambda, \gamma, \alpha, \beta)$, *then*

$$|a_2| \le \frac{|\gamma||\eta|}{([2]_q-1)\,|1-\alpha||b_2|} \tag{12}$$

and

$$|a_n| \le \frac{|\gamma||\eta|}{([n]_q-1)\,|1-\alpha||b_n|}\prod_{j=2}^{n-1}\left(1+\frac{|\gamma||\eta|}{([j]_q-1)\,|1-\alpha|}\right),\quad n \ge 3, \tag{13}$$

where

$$\eta = (1-\beta)\cos\lambda + i(1-\alpha)\sin\lambda. \tag{14}$$

Proof. Let $f(z) \in \mathcal{ST}_q(g, \lambda, \gamma, \alpha, \beta)$. Then by Theorem 2.1, we have

$$\operatorname{Re}\left\{e^{i\lambda}\left(1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)\right)\right\} > \frac{\beta-\alpha}{1-\alpha}\cos\lambda,\ \ (z\in\mathcal{U}). \tag{15}$$

Let us define $p(z)$ by

(16)
$$e^{i\lambda}\left(1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)\right) = \left[\left(\frac{1-\beta}{1-\alpha}\right)p(z)+\left(\frac{\beta-\alpha}{1-\alpha}\right)\right]\cos\lambda + i\sin\lambda.$$

Then $p(z)$ is anlaytic in $\mathcal{U}$ with $p(0) = 1$ and $\operatorname{Re} p(z) > 0, z \in \mathcal{U}$. Let

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n, \quad z \in \mathcal{U}. \tag{17}$$

Then (16) becomes

$$1-\frac{2}{\gamma}+\frac{2}{\gamma}\left(\frac{zD_q(f*g)(z)}{(f*g)(z)}\right)=1+\frac{(1-\beta)\cos\lambda+\mathrm{i}(1-\alpha)\sin\lambda}{\mathrm{e}^{\mathrm{i}\lambda}(1-\alpha)}\sum_{n=1}^{\infty}p_nz^n. \tag{18}$$

That is,

$$2\mathrm{e}^{\mathrm{i}\lambda}(1-\alpha)\left[zD_q(f*g)(z)-(f*g)(z)\right]=\gamma\eta(f*g)(z)\left(\sum_{n=1}^{\infty}p_nz^n\right), \tag{19}$$

where η is given by (14). Using (3) in (19), we obtain

$$2\mathrm{e}^{\mathrm{i}\lambda}(1-\alpha)\left[\sum_{n=2}^{\infty}\left([n]_q-1\right)a_nb_nz^n\right]=\gamma\eta\left[z+\sum_{n=2}^{\infty}a_nb_nz^n\right]\left(\sum_{n=1}^{\infty}p_nz^n\right). \tag{20}$$

Comparing coefficients of z^n on both sides,

$$2\mathrm{e}^{\mathrm{i}\lambda}(1-\alpha)\left([n]_q-1\right)a_nb_n=\gamma\eta\left(p_{n-1}+a_2b_2p_{n-2}+\cdots+a_{n-1}b_{n-1}p_1\right). \tag{21}$$

Taking absolute on bothsides and then applying the coefficient estimates $|p_n|\leq 2$ for Caratheodory functions [4], we obtain

$$|a_n|\leq\frac{|\gamma||\eta|}{([n]_q-1)|1-\alpha||b_n|}\left(1+|a_2||b_2|+\cdots+|a_{n-1}||b_{n-1}|\right). \tag{22}$$

For $n=2$,

$$|a_2|\leq\frac{|\gamma||\eta|}{([2]_q-1)|1-\alpha||b_2|},$$

which proves (12).

For $n=3$,

$$|a_3|\leq\frac{|\gamma||\eta|}{([3]_q-1)|1-\alpha||b_3|}\left(1+\frac{|\gamma||\eta|}{([2]_q-1)|1-\alpha|}\right).$$

Therefore, (13) holds for $n=3$.

Assume that (13) is true for $n=k$.

Consider,

$$\begin{aligned}|a_{n+1}|\leq\frac{|\gamma||\eta|}{([n+1]_q-1)|1-\alpha||b_{n+1}|}&\left\{1+\frac{|\gamma||\eta|}{([2]_q-1)|1-\alpha|}\right.\\&+\frac{|\gamma||\eta|}{([3]_q-1)|1-\alpha|}\left(1+\frac{|\gamma||\eta|}{([2]_q-1)|1-\alpha|}\right)+\cdots\\&\left.+\frac{|\gamma||\eta|}{([n]_q-1)|1-\alpha|}\prod_{j=2}^{n-1}\left(1+\frac{|\gamma||\eta|}{([j]_q-1)|1-\alpha|}\right)\right\}\end{aligned}$$

$$= \frac{|\gamma||\eta|}{([n+1]_q - 1)|1-\alpha||b_{n+1}|} \prod_{j=2}^{n} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)\,|1-\alpha|}\right).$$

Therefore, the result is true for $n = k+1$, using mathematical induction, (13) holds true for all $n \geq 3$. □

The error function erf defined by [1]

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)\mathrm{d}t = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)n!}, \tag{23}$$

is the subject of intensive studies and applications during the last years. Several properties and inequalities of error function can be found in [3, 9]. In [10] the authors study the properties of complementary error function occurring widely in almost every branch of applied mathematics and mathematical physics, e.g., probability and statistics [8] and data analysis [13]. Its inverse, introduced by Carlitz [7], which we will denote by $inverf$, appears in multiple areas of mathematics and the natural sciences.

Let $E_r f$ (see [25]) be a normalized analytic function which is obtained from (23), and given by

$$E_r f = z + \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(2n-1)(n-1)!} z^n.$$

Let $g(z) = E_r f$ in Theorem 2.2, then we get the following corollary.

Corollary 2.3. *If $f(z) \in \mathcal{ST}_q(E_r f, \lambda, \gamma, \alpha, \beta)$, then $|a_2| \leq \frac{3|\gamma||\eta|}{(1-[2]_q)|1-\alpha|}$ and*

$$|a_n| \leq \frac{|\gamma||\eta|(2n-1)(n-1)!}{(-1)^{n-1}([n]_q - 1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)|1-\alpha|}\right), \quad n \geq 3.$$

If $g(z) = \dfrac{z}{1-z}$ in Theorem 2.2, then we get the following corollary.

Corollary 2.4. *If $f(z) \in \mathcal{ST}_q\left(\dfrac{z}{1-z}, \lambda, \gamma, \alpha, \beta\right)$, then $|a_2| \leq \frac{|\gamma||\eta|}{([2]_q-1)|1-\alpha|}$ and*

$$|a_n| \leq \frac{|\gamma||\eta|}{([n]_q - 1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)|1-\alpha|}\right), \quad n \geq 3.$$

If $q \to 1$ in the above Corollary 2.4, we get the following corollary.

Corollary 2.5. *If $f(z) \in \mathcal{ST}\left(\dfrac{z}{1-z}, \lambda, \gamma, \alpha, \beta\right)$, then $|a_2| \leq \frac{|\gamma||\eta|}{|1-\alpha|}$ and*

$$|a_n| \leq \frac{|\gamma||\eta|}{(n-1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{(j-1)|1-\alpha|}\right), \quad n \geq 3.$$

Corollary 2.6. *If we put $\lambda = 0, \gamma = 2$ in the Corollary 2.5, we get the Theorem 2.3 in [20] which states that. If $f(z) \in \mathcal{ST}(\alpha, \beta)$, then $|a_2| \leq \frac{2(1-\beta)}{|1-\alpha|}$ and*

$$|a_n| \leq \frac{2(1-\beta)}{(n-1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{2(1-\beta)}{(j-1)|1-\alpha|}\right), \;\; n \geq 3.$$

Corollary 2.7. *Let $\alpha = 0$ in Corollary 2.6. Then we get*

$$|a_n| \leq \frac{1}{(n-1)!} \prod_{j=2}^{n} (j - 2\beta), \;\; n \geq 2,$$

a result by Roberston [26].

Theorem 2.8. *Let $f(z) \in \mathcal{KV}_q(g, \lambda, \gamma, \alpha, \beta)$ with $0 \leq \alpha \leq \beta$. Then $f(z) \in \mathcal{KV}_q\left(g, \lambda, \gamma, 0, \frac{\beta-\alpha}{1-\alpha}\right)$.*

Theorem 2.9. *If $f(z) \in \mathcal{KV}_q(g, \lambda, \gamma, \alpha, \beta)$, then*

$$|a_2| \leq \frac{|\gamma||\eta|}{[2]_q\left([2]_q - 1\right)|1-\alpha||b_2|}, \tag{24}$$

$$|a_n| \leq \frac{|\gamma||\eta|}{[n]_q\left([n]_q - 1\right)|1-\alpha||b_n|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)|1-\alpha|}\right), \;\; n \geq 3. \tag{25}$$

If $g(z) = E_r f$ in Theorem 2.9, then we get the following corollary.

Corollary 2.10. *If $f(z) \in \mathcal{KV}_q(E_r f, \lambda, \gamma, \alpha, \beta)$, then*

$$|a_2| \leq \frac{3|\gamma||\eta||}{[2]_q(1-[2]_q)|1-\alpha|},$$

$$|a_n| \leq \frac{|\gamma||\eta|(2n-1)(n-1)!}{(-1)^{n-1}[n]_q\left([n]_q - 1\right)|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)|1-\alpha|}\right), \;\; n \geq 3.$$

If $g(z) = \dfrac{z}{1-z}$ in Theorem 2.9, then we get the following corollary.

Corollary 2.11. *If $f(z) \in \mathcal{KV}_q\left(\dfrac{z}{1-z}, \lambda, \gamma, \alpha, \beta\right)$, then*

$$|a_2| \leq \frac{|\gamma||\eta|}{[2]_q([2]_q - 1)|1-\alpha|},$$

$$|a_n| \leq \frac{|\gamma||\eta|}{[n]_q\left([n]_q - 1\right)|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{([j]_q - 1)|1-\alpha|}\right), \;\; n \geq 3.$$

If $q \to 1$ in the Corollary 2.11, we get the following corollary.

Corollary 2.12. *If* $f(z) \in \mathcal{KV}\left(\dfrac{z}{1-z}, \lambda, \gamma, \alpha, \beta\right)$, *then* $|a_2| \leq \frac{|\gamma||\eta|}{2|1-\alpha|}$ *and*

$$|a_n| \leq \frac{|\gamma||\eta|}{n\,(n-1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{|\gamma||\eta|}{(j-1)|1-\alpha|}\right), \ \ n \geq 3.$$

Corollary 2.13. *If we put* $\lambda = 0, \gamma = 2$ *in the Corollary* 2.12, *we get the Corollary* 2.5 *of* [20] *which states that. If* $f(z) \in \mathcal{KV}\,(\alpha, \beta)$, *then* $|a_2| \leq \frac{1-\beta}{|1-\alpha|}$ *and*

$$|a_n| \leq \frac{2(1-\beta)}{n\,(n-1)\,|1-\alpha|} \prod_{j=2}^{n-1} \left(1 + \frac{2(1-\beta)}{(j-1)|1-\alpha|}\right), \ \ n \geq 3.$$

Remark 2.14. If we let $\alpha = 0$ in Corollary 2.13, we get a result given by Roberston [26]: $|a_n| \leq \frac{1}{n!} \prod\limits_{j=2}^{n} (j - 2\beta), \ \ n \geq 2.$

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