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ON THE NON-COMMUTING GRAPH OF A GROUP

BEHNAZ TOLUE

Abstract. In this paper, groups whose non-commuting graphs are k-apex for $1 \leq k \leq 5$ are classified. The 1-planarity of the non-commuting graph for an AC-group G is discussed. Moreover, the k-connectivity of the non-commuting graph is verified, for $k \leq 6$. Finally, some properties of the line graph of the non-commuting graph of a group are studied.

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1. INTRODUCTION

Graphs can be assigned to algebraic structures in many different ways. One of such graphs is the non-commuting graph associated to a group [1, 8]. Let G be a non-abelian group. The non-commuting graph Γ_G associated to G is a graph whose vertices are non-central elements of G and two distinct vertices join by an edge if they do not commute.

Recall that a graph Γ is k-apex, if there exist t vertices $v_1, v_2, \cdots, v_t$ of the graph Γ such that the induced graph $\Gamma - \{v_1, \cdots, v_t\}$ is planar, where $t \leq k$ is a positive integer. In the k-Apex problem the task is to find at most k vertices whose deletion makes the given graph planar. In other words, for a given graph Γ and a parameter k, the k-apex-ness is to decide whether deleting at most k vertices from Γ can result in a planar graph. Such a set of vertices is sometimes called a set of apex vertices or apices. Let us denote 1-apex graph by apex graph.

Let $D_{2n} = \langle a, b : a^n = b^2 = 1, a^b = a^{-1} \rangle$ be the dihedral group of order $2n$, $n \geq 4$. In Section 2, we investigate the k-apex non-commuting graphs, $1 \leq k \leq 5$. The non-commuting graph Γ_G is 5-apex if and only if G is symmetric group S_3, dihedral groups D_8, D_{10}, D_{12}, D_{14}, quaternion group Q_8 or $T = \langle a, b : a^6 = 1,\ b^2 = a^3,\ a^b = a^{-1} \rangle$. In the process of proving the last result, the apex-ness of the non-commuting graphs of the groups of order less than 21 is achieved.

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A graph is called 1-planar if it can be drawn in the plane such that each edge is crossed at most once. Czap and Hudàk present the full characterization of 1-planar complete k-partite graphs [3]. A non-abelian group G is an AC-group, if all the centralizers of its non-central elements are abelian. By Theorem 2.11 in [11], we know the non-commuting graph associated to an AC-group is complete s-partite graph, where $s+1$ is the number of distinct centralizers. All these considerations imply that all AC-groups whose non-commuting graphs are 1-planar, are S_3, D_8, and Q_8.

Let W be a set of vertices. If $\Gamma - W$ is not connected, then W separates Γ and W is called a vertex-separator. For any $k \geq 1$, Γ is k-connected if it has the order at least $k+1$ and no set of $k-1$ vertices is a separator. We discuss the separator set of the non-commuting graph of an AC-group. Moreover, we observe that Γ_G is not k-connected, for $k = 1, 2$. The non-commuting graph $\Gamma_{D_{2p}}$ is p-connected, where p is an odd prime number.

In the third section, we observe that the line graph of the non-commuting graph $L(\Gamma_G)$ is a connected graph with a hamiltonian cycle. It is not planar. Furthermore, we study its domination, clique and independence number.

2. 5-APEX AND 1-PLANAR NON-COMMUTING GRAPHS

For positive integers l, r and t, let $K_{l[r]}$ denote a complete l-partite graph with each part of order r and let $K_{l[r],t}$ denote a complete (l + 1)-partite graph with l parts of order r and a part of order t (cf. [13]).

An example of a large class of AC-groups is given by the dihedral groups. The structure of the non-commuting graph associated to dihedral group D_{2n} depends on the integer n. If n is an odd number, then it is a complete $(n+1)$-partite graph with $(n+1)$ parts $\{a, a^2, \cdots, a^{n-1}\}$, $\{b\}$, $\{ab\}, \cdots, \{a^{n-1}b\}$. It is not hard to deduce that by omitting the vertices $a^2b, \cdots, a^{n-1}b$ the remaining graph is planar, so $\Gamma_{D_{2n}}$ is $(n-2)$-apex for odd n. The other graph parameters can be obtained too. For instance, $\Gamma_{D_{2n}}$ is n-connected, where n is an odd number. Note that if we omit n vertices $b, ab, \cdots, a^{n-1}b$, then $\Gamma_{D_{2n}}$ is not connected.

Now, assume n is an even integer. Therefore, $\Gamma_{D_{2n}}$ is complete $(\frac{n}{2}+1)$-partite graph. The $(\frac{n}{2}+1)$ parts are $\{a, a^2, \cdots, a^{\frac{n}{2}-1}, a^{\frac{n}{2}+1}, \cdots, a^{n-1}\}$ and $\{a^i b, a^{i+\frac{n}{2}} b\}$, $0 \leq i \leq \frac{n}{2} - 1$. If $n = 4$, then $\Gamma_{D_8} = K_{3[2]}$ which is planar and 4-connected. Let $n \geq 6$. Omit $(n-2)$ of the vertices which are not the powers of a, say $\{a^i b, a^{i+\frac{n}{2}} b\}$, $1 \leq i \leq \frac{n}{2} - 1$. Hence, $\Gamma_{D_{2n}}$ is $(n-2)$-apex for even n. Moreover, it is n-connected, while the n vertices $a^i b, a^{i+\frac{n}{2}} b$ are vertex separators, $0 \leq i \leq \frac{n}{2} - 1$ (for the structure of $\Gamma_{D_{2n}}$, see [13, Lemma 2.3]). We conclude the following result.

Proposition 2.1. *The non-commuting graph of D_{2n} is $(n-2)$-apex and n-connected, for $n \geq 4$.*

Let A_4 be the alternating group on 4 letters, then $\Gamma_{A_4} = K_{4[2],3}$ and clearly 6-apex and 8-connected. The proof of Theorem 2.2 is inspired by the proof of [1, Proposition 2.3].

THEOREM 2.2. *The non-commuting graph Γ_G is apex if and only if $G \cong S_3,\ D_8,\ Q_8$.*

Proof. Suppose Γ_G is apex. By definition, there is $v \in V(\Gamma_G)$ such that $\Gamma_G - \{v\}$ is planar and so we have for the clique number $\omega(\Gamma_G - \{v\}) < 5$. We have three cases for the vertex v.

(i) The vertex v commutes with all other vertices of Γ_G.
(ii) The vertex v does not commute with some vertices of Γ_G.
(iii) The vertex v does not commute with all other vertices of Γ_G.

We claim that the first case does not occur, since non-commuting graphs are connected (see [1, Proposition 2.1]). If (ii) or (iii) hold, then $\omega(\Gamma_G) < 6$. Thus, $G/Z(G)$ is a finite group, by the main result of [9]. Clearly, the size of the set $G - Z(G) - \{v\}$ is greater than 2. There are $x, y \in G - Z(G) - \{v\}$ such that $[x, y] \neq 1$, because otherwise Γ_G is planar and the non-commuting graph of S_3, D_8 or Q_8 does not have the same figure as in this case explained. We prove that $|Z(G)| \leq 5$. If $Z \subseteq Z(G)$ and $|Z| > 5$, then the induced subgraph Δ of $\Gamma_G - \{v\}$ on the vertices $xZ \cup yZ$ is not planar and we get a contradiction. Hence G is finite and so Γ_G is a finite graph. Therefore, there is a vertex $t \in G - Z(G) - \{v\}$ such that $\deg_{\Gamma_{G-\{v\}}}(t) \leq 5$ (see e.g. [2, Corollary 3.5.9]). Two cases can happen $[t, v] = 1$ or $[t, v] \neq 1$. Thus $|G| - |C_G(t)| \leq 5$ or $|G| - |C_G(t)| \leq 6$. So, by the fact that $|C_G(t)| \leq |G|/2$, we conclude that $|G| \leq 12$ and $G \cong S_3,\ D_8,\ Q_8,\ D_{10},\ D_{12},\ T = \langle a, b : a^6 = 1,\ b^2 = a^3,\ a^b = a^{-1}\rangle$, and A_4. By the argument before the theorem, Γ_G is planar for $S_3,\ D_8,\ Q_8$, while $\Gamma_{D_{10}} = K_{5[1],4}$ is 3-apex, $\Gamma_{D_{12}} = K_{3[2],4}$ is 4-apex and $\Gamma_{A_4} = K_{4[2],3}$ is 6-apex. By the presentation of T, we deduce that $\Gamma_T \cong K_{3[2],4}$ and 4-apex. □

As in the proof of Theorem 2.2, we can find the groups whose non-commuting graphs are k-apex. Let Γ_G be k-apex. So, $\omega(\Gamma_G - \{v_1, v_2, \cdots, v_k\}) < 5$, where any v_i are vertices that, by omitting them, the remaining graph is planar $1 \leq i \leq k$. By computations and the fact that G is a non-abelian group, we have at least two elements $x, y \in G - Z(G) - \{v_1, v_2, \cdots, v_k\}$. If $[x, y] \neq 1$, then $|Z(G)| \leq 5$ and Γ_G is a finite graph. Therefore, for the vertex $t \in G - Z(G) - \{v_1, v_2, \cdots, v_k\}$, we have $|G| - |C_G(t)| \leq 5 + k$, in the worst situation. This implies that $|G| \leq 2(5 + k)$. If for all vertices $x, y \in G - Z(G) - \{v_1, v_2, \cdots, v_k\}$ and x and y commute, then $\deg(x) = |G| - |C_G(x)| \leq k$ and so $|G| \leq 2k$. So, we shall investigate the groups with the order less than $2(5 + k)$. Hence, we have the following result.

THEOREM 2.3. *Let Γ_G be the non-commuting graph associated to the non-abelian group G.*

(i) *Γ_G is 2-apex if and only if $G \cong S_3,\ D_8,\ Q_8$.*

(ii) Γ_G *is* 3-*apex if and only if* $G \cong S_3,\ D_8,\ Q_8,\ D_{10}$.
(iii) Γ_G *is* 4-*apex if and only if* $G \cong S_3,\ D_8,\ Q_8,\ D_{10},\ D_{12},\ T$.
(iv) Γ_G *is* 5-*apex if and only if* $G \cong S_3,\ D_8,\ Q_8,\ D_{10},\ D_{12},\ T,\ D_{14}$.

Proof. According to the arguments before the theorem, we check the assertion for all non-abelian groups with $|G| \leq 20$. Using GAP [5], we observe that all these groups are AC-groups. Therefore, again [11, Theorem 2.1] implies that Γ_G is a complete s-partite graph. For instance, the only non-abelian group of order 14 is D_{14}, $\Gamma_{D_{14}} = K_{7[1],6}$. If we omit 5 vertices in the singleton parts, then the graph is planar. There are nine non-abelian groups of order 16. The non-commuting graph associate to 6 of them are $K_{3[4]}$ and so it is 6-apex. Moreover, 3 of them are $K_{4[2],6}$ and so 6-apex. There are 3 non-abelian groups of order 18. The non-commuting graph of two of them are $K_{9[1],8}$ and 7-apex, while the non-commuting graph of the other one is $K_{3[3],6}$ and 7-apex. Finally, there are 3 non-abelian group of order 20. The non-commuting graph of two of them is $K_{5[2],8}$ and the other one is $K_{5[3],4}$, which are 8-apex and 13-apex, respectively. □

Note that from the results of [3] it follows that: if a graph Γ contains one of the graphs $K_{7,3},\ K_{5,4},\ K_{4,3,1},\ K_{2[3],2}$ or $K_{4[1],3}$, then Γ is not 1-planar. Moreover, if a non-1-planar graph contains a complete multipartite graph, then it contains at least one from the list above.

THEOREM 2.4. *Let G be an AC-group. The non-commuting graph Γ_G is* 1-*planar if and only if* $G \cong S_3,\ D_8,\ Q_8$.

Proof. Suppose Γ_G is 1-planar. Since G is an AC-group, again from [11, Theorem 2.11] we deduce that Γ_G is a complete s-partite graph. Now, 1-planarity of the graph implies that it does not contain K_7 (see [4] for more details). This means $\omega(\Gamma_G) < 7$. Similar arguments to those in the proof of Theorem 2.2 imply that $G/Z(G)$ is finite. We claim that $|Z(G)| < 4$, because otherwise if $Z \subset Z(G)$ and $|Z| \geq 4$, then the induced subgraph Γ_0 on $xZ \cup yZ$ is not 1-planar as it contains $K_{4[1],3}$, where x, y are two elements of the non-abelian group G that $[x, y] \neq 1$. From this fact it follows that Γ_G is a finite graph. Since Γ_G is 1-planar, there is a vertex like x such that $\deg(x) = |G| - |C_G(x)| \leq 7$. We conclude that $|G| \leq 14$. We know that $\Gamma_{D_{10}} \cong K_{5[1],4}$, $\Gamma_{D_{14}} \cong K_{7[1],6}$ $\Gamma_{A_4} \cong K_{4[2],3}$ all contain $K_{4[1],3}$ and so they are not 1-planar. Moreover, $\Gamma_{D_{12}} \cong \Gamma_T \cong K_{3[2],4}$ contain $K_{4,3,1}$ and are not 1-planar. □

A graph is outer-planar if it does not contain the subdivisions $K_{2,3}$ and K_4. By a similar argument to that in Theorem 2.4, we deduce that Γ_G is an outer-planar graph if and only if G is isomorphic to $S_3,\ D_8$ or Q_8.

There is no complete non-commuting graph, so if Γ_G is a non-commuting graph associated to an AC-group G, then it is a complete s-partite graph [11, Theorem 2.11] and at least there is a part with more than one vertex.

PROPOSITION 2.5. *If G is an AC-group, then Γ_G is k-connected, where $k \geq s-1$ and s is the number of centralizers of distinct non-central elements.*

Proof. By the argument before the theorem, the worst case is when Γ_G is a complete s-partite graph with just one part with more than one vertex. If we choose $s-1$ vertices of the parts with one vertex, then we obtain a separator set. □

Let Γ_G be k-connected. Suppose, by omitting $\{v_1, \cdots, v_k\}$, there is no path between two vertices x and y. The vertices x and y join all to v_i, because otherwise we can find a separator set with the size less than k and we get a contradiction. Therefore, $\deg(x), \deg(y) \geq k$. Γ_{S_3} is 3-connected, the non-commuting graph of D_8 or Q_8 is 4-connected and the non-commuting graph of D_{12}, T or A_4 is 6-connected.

3. THE LINE GRAPH OF THE NON-COMMUTING GRAPH Γ_G

Let us denote the line graph of the non-commuting graph Γ_G by $L(\Gamma_G)$. Clearly, $L(\Gamma_G)$ is a graph with edges of Γ_G as its vertices and $e_i = \{x_i, y_i\}$ and $e_j = \{x_j, y_j\}$ join if $e_i \cap e_j \neq \emptyset$. Clearly, $\deg(e_i) = 2|G| - |C_G(x_i)| - |C_G(y_i)| - 2$. If $e_i = \{x_i, y_i\}$ is an isolated vertex in $L(\Gamma_G)$, then it does not have any common vertex with the other edges, say $e = \{x, y\}$. As $\operatorname{diam}(\Gamma_G) = 2$, there is a path between x and x_i of length less than 2. But this fact shows there is an edge for which one of its ends is the same as that of e_i. Thus, there is no isolated vertex in $L(\Gamma_G)$.

PROPOSITION 3.1. *For line graph of the non-commuting graph we have* $\operatorname{diam}(L(\Gamma_G)) \leq 3$ *and* $\operatorname{girth}(L(\Gamma_G)) = 3$.

Proof. Let $e_i = \{x_i, y_i\}$ and $e_j = \{x_j, y_j\}$ be two non-adjacent vertices of $L(\Gamma_G)$. Without loss of generality, we concentrate on one pair of end points x_i and x_j. Since $\operatorname{diam}(\Gamma_G) = 2$, $d(x_i, x_j) \leq 2$, if $e_{ij} = \{x_i, x_j\}$, then $d(e_i, e_j) = 2$. Suppose $d(x_i, x_j) = 2$, $e_{ik} = \{x_i, x_k\}$ and $e_{kj} = \{x_k, x_j\}$. Hence, the rest follows clearly. □

Two subgraphs Δ_1 and Δ_2 are said to be close in the graph Γ if they are disjoint and there is an edge of Γ joining a vertex of Δ_1 and one of Δ_2. If Δ_1 and Δ_2 are disjoint and not close, then Δ_1 and Δ_2 are remote. The degree of an edge in the graph Γ is denoted by $\deg_\Gamma(e)$, which is the number of vertices of Γ close to e. A cycle ζ of the graph Γ is called a dominating cycle (D-cycle) if every edge of G is incident with at least one vertex of ζ.

PROPOSITION 3.2. *The non-commuting graph Γ_G is D-cyclic.*

Proof. The non-commuting graph is not a tree and by the definition of the degree of an edge. For any edge e, we have

$$\deg_{\Gamma_G}(e) = 2|G| - |C_G(x)| - |C_G(y)| - 2,$$

where x and y are two end points of the edge e. Thus,

$$|G| - |Z(G)| - 2 \leq 2|G| - |C_G(x)| - |C_G(x')| - 2 \leq \deg_{\Gamma_G}(e) + \deg_{\Gamma_G}(f),$$

where e and f are remote edges and x' is an end point of f. Hence, by [12, Theorem 2.], the result follows. □

Harary and Nash-Williams [7] showed that the existence of a dominating cycle in Γ is essentially equivalent to the existence of a hamiltonian cycle in the line graph of Γ, denoted $L(\Gamma)$. Therefore, there is a hamiltonian cycle for the line graph of the non-commuting graph Γ_G.

Greenwell and Hemminger [6] proved that a graph Γ has a planar line graph if and only if Γ has no subdivision isomorphic to $K_{3,3}$, $K_{1,5}$, P_4+K_1 or $K_2+\overline{K_3}$.

Theorem 3.3. *The line graph of the non-commuting graph of the group G is not planar.*

Proof. Suppose $L(\Gamma_G)$ is planar. Therefore, Γ_G does not contain a subdivision isomorphic to $K_{1,5}$ and the degree of vertices are less than equal to 4. By the fact that degree of each vertex v of the non-commuting graph is $|G| - |C_G(v)|$, we deduce the possible non-abelian groups are S_3, D_8 or Q_8. But the diagram of Γ_{S_3} includes the subdivision $K_2 + \overline{K_3}$, while the diagram of $\Gamma_{D_8} \cong \Gamma_{Q_8}$ contains $K_{3,3}$. □

The size of the largest complete induced subgraph of the graph Γ is called the clique number and is denoted by $\omega(\Gamma)$.

Proposition 3.4. *The clique number of the line non-commuting graph is $max\{|G| - |C_G(x)| : x \in V(\Gamma_G)\}$.*

Proof. For every vertex $x \in V(\Gamma_G)$, there are $|G|-|C_G(x)|$ vertices in $L(\Gamma_G)$. All these vertices have x in common, so they are adjacent and form a clique for $L(\Gamma_G)$. Hence, the assertion is clear. □

As a consequence of the above proposition, $\omega(L(\Gamma_{S_3})) = 4$ and $\omega(L(\Gamma_{D_{2n}}))$ is $2n-4$ or $2n-2$, for even or odd integer n, respectively.

The subset S of vertices of the graph Γ is a dominating set if all the vertices outside of S join to at least one of the inside vertices of S. The size of the smallest dominating set is called domination number and is denoted by $\gamma(\Gamma)$.

Proposition 3.5. *For a non-abelian group G, $\gamma(L(\Gamma_G)) \geq 2$.*

Proof. It is enough to prove $L(\Gamma_G)$ does not have a singleton dominating set. If $S = \{e\}$ is a dominating set for $L(\Gamma_G)$, then e dominate all other vertices of $L(\Gamma_G)$, where $e = \{x, y\}$. This means for any vertex $f_i \in V(L(\Gamma_G))$, e and f_i have a vertex in common. Without lose of generality, suppose $f_1 = \{y, t_1\}$ and $f_2 = \{y, t_2\}$. The vertices t_1, t_2 are not adjacent, because otherwise an edge formed by them is not dominated by e. By a similar argument, t_i's are not adjacent to the vertices of $N(x)$, where $N(x)$ is the set of all neighbors of

x, $i = 1, 2$. On the other hand, there is no vertex of degree one in the non-commuting graph, which implies that t_1, t_2 both join to x. One can imagine the diagram of the graph in triangles that have one side in common. Thus, Γ_G is planar and accordingly $G \cong S_3$, D_8, Q_8. But the diagram of the non-commuting graph of these groups does not match to what we observed. □

The above bound is sharp, $\gamma(L(\Gamma_{S_3})) = 2$. The non-commuting graph is a connected graph and $d(x, y) \leq 2$, for all pairs of vertices x and y. This fact is used in Lemma 3.6 (period).

Lemma 3.6. *Let $e = \{x, y\}$ be a vertex of the line graph $L(\Gamma_G)$. Then the independent set which contains the vertex e has at least $\sum_{i=1}^{l} \deg(t_i)$ elements, where t_i's are the vertices whose distance to x, y is 2 and l is the number of them. Moreover, $l = |C_G(x) \cap C_G(y)| - |Z(G)|$.*

Proof. If t is a vertex such that $d(t, x) = d(t, y) = 2$, then there is a vertex $s \in N(x)$ and $\{s, t\}$ is an edge which does not have a common vertex with e. Since the degree of vertices in the non-commuting graph is larger than 2 and t does not join to x and y directly, $\{t, s_i\}$ and e are non-adjacent edges, where $s_i \in N(t)$. Thus, for every vertex of distance 2 to x, y, there are $\deg(t)$ edges that are independent of e, which means there are $\deg(t)$ edges without the common vertices with e. Hence, the first part is clear. As l is the number of vertices whose distance to x, y is 2, we should count the vertices that commute with x, y. □

We denote the size of the independent set of the graph Γ that includes the vertex x by $\alpha_x(\Gamma)$ and the independence number of the graph by $\alpha(\Gamma)$. Therefore, with the notation from Lemma 3.6, we have $\alpha_e(L(\Gamma_G)) \geq \sum_{i=1}^{l} \deg(t_i)$.

Theorem 3.7. *Let G be an AC-group. There is no independent vertex of $L(\Gamma_G)$ for an arbitrary vertex $e = \{x, y\}$.*

Proof. By Lemma 3.6, it is enough to consider vertices of Γ_G such that their distance to both x, y are 2. Let t be a vertex of Γ_G such that $d(t, x) = d(t, y) = 2$. Then $t \in C_G(x) \cap C_G(y) - Z(G)$. By the property of AC-groups, $[x, y] = 1$ (see [10, Lemma 3.2]), which gives a contradiction. □

For the line graph of the non-commuting graph associated to an AC-group, $\alpha(L(\Gamma_G)) = 1$.

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Hakim Sabzevari University
Department of Pure Mathematics
Sabzevar, Iran
E-mail: b.tolue@gmail.com
b.tolue@hsu.ac.ir