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SOME WEIGHTED INEQUALITIES OF CHEBYSHEV TYPE VIA RL-APPROACH

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Abstract. By using the Riemann-Liouville fractional integral operator, we establish new weighted results of Chebyshev inequality type. Other integral inequalities of fractional order are also proved. Some classical results can be deduced as special cases.

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1. INTRODUCTION

We begin this paper by considering the well known Chebyshev functional [4]:

$$T(f,g) := \frac{1}{b-a}\int_a^b f(x)g(x)\mathrm{d}x - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \times \frac{1}{b-a}\int_a^b g(x)\mathrm{d}x, \tag{1}$$

where f and g are two integrable functions on $[a,b]$.

If f and g are monotonic in the same direction on $[a,b]$, it is known that $T(f,g) \geq 0$.

In the case f is bounded by some real constants m and M, and g is absolutely continuous with $g' \in L^{\infty}[a,b]$, it has been proved (see [13]) that:

$$|T(f,g)| \leq \frac{b-a}{8}(M-m)||g'||_{\infty}. \tag{2}$$

Recently, P. Cerone and S. S. Dragomir [3] proved that, if f and g are absolutely continuous on $[a,b]$, with $f', g' \in L^{\infty}[a,b]$, the inequality

$$|T(f,g)| \leq \frac{1}{12}||f'||_{\infty}||g'||_{\infty}(b-a)^2 \tag{3}$$

is valid. Very recently, by considering the weighted Chebyshev functional [4]:

$$T(f,g,p) := \int_a^b p(x)\int_a^b p(x)f(x)g(x)\mathrm{d}x - \int_a^b p(x)f(x)\mathrm{d}x \times \int_a^b p(x)g(x)\mathrm{d}x, \tag{4}$$

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K. M. Awan et al. proved the following important result [1]: if ϕ is an absolutely continuous function on $[a,b]$ and p is a positive and integrable function on $[a,b]$, with $(\phi')^2 \in L^1[a,b]$, the following inequality is valid:

$$T(\phi,\phi,p) \leq \frac{1}{P^2(b)} \int_a^b \tilde{P}(x) \left(\phi'\right)^2 (x)\mathrm{d}x, \tag{5}$$

where $P(x) = \int_a^x p(t)\mathrm{d}t$ and $\tilde{P}(x) = P(x)\int_a^b tp(t)\mathrm{d}t - P(b)\int_a^x tp(t)\mathrm{d}t$.

Many researchers have been concerned with the functionals (1) and (4). For more details, we refer to [3, 5, 6, 7, 8, 9, 11, 12] and the references therein.

The main purpose of this paper is to establish some new inequalities for (1) and (4) by using the Riemann-Liouville fractional integrals. We generalize some results related to the weighted Chebyshev functional. Other classes of the Chebyshev inequalities are also obtained as special cases. Our results have some relationships with those obtained in the good paper [1].

2. PRELIMINARIES

Definition 2.1. The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, for a continuous function f on $[a,b]$, is defined as

$$\begin{aligned} &J_a^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau,\ \alpha > 0,\ a < t \leq b, \\ &J_a^0 f(t) = f(t), \end{aligned} \tag{6}$$

We remark that for $\alpha > 0$, $\beta > 0$, we have the following property

$$J_a^\alpha J_a^\beta f(t) = J_a^{\alpha+\beta} f(t), \tag{7}$$

which implies

$$J_a^\alpha J_a^\beta f(t) = J_a^\beta J_a^\alpha f(t). \tag{8}$$

For more details, one can consult [10].

3. MAIN RESULTS

We begin by proving the following theorem, using some ideas from [1].

Theorem 3.1. *Suppose that $\phi : [a,b] \to \mathbb{R}$ is an absolutely continuous function and $p : [a,b] \to \mathbb{R}^+$ is an integrable function. If $(\phi')^2 \in L^1[a,b]$, then for all $\alpha > 0$, we have*

$$J_a^\alpha p(b) J_a^\alpha p\phi^2(b) - \left(J_a^\alpha p\phi(b)\right)^2 \leq \int_a^b H(x) \left(\phi'(x)\right)^2 \mathrm{d}x, \tag{9}$$

with

$$\begin{aligned} H(x) := \frac{1}{2\Gamma(\alpha)} \Bigg[\left(J_a^\alpha bp(b) \int_a^x (b-t)^{\alpha-1} p(t)\mathrm{d}t \right) - J_a^\alpha p(b) \\ \times \int_a^x t(b-t)^{\alpha-1} p(t)\mathrm{d}t \Bigg]. \end{aligned} \tag{10}$$

Proof. We have

$$\begin{aligned} & J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b) \\ & = \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \left[(f(s)-f(t))\right. \\ & \qquad \left. \times\ (g(s)-g(t))\right] \mathrm{d}s\mathrm{d}t. \end{aligned} \tag{11}$$

Therefore,

$$\begin{aligned} & J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b) \\ & = \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \\ & \qquad \times \left[(f(s)-f(t)) \left(\int_t^s g'(x)\mathrm{d}x\right)\right] \mathrm{d}s\mathrm{d}t. \end{aligned} \tag{12}$$

Since $a \leq t \leq x \leq s \leq b$, we can write

$$\begin{aligned} & J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b) \\ & = \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^x (b-t)^{\alpha-1} p(t) \int_a^b (b-s)^{\alpha-1} \left(f(s)-f(t)\right) \\ & \qquad \times p(s)\mathrm{d}s\mathrm{d}t \left(g'(x)\right) \mathrm{d}x. \end{aligned} \tag{13}$$

Taking $f(x) = x$, we obtain

$$\begin{aligned} & J_a^\alpha p(b) J_a^\alpha b\,(pg)\,(b) - J_a^\alpha bp(b) J_a^\alpha pg(b) \\ & = \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^x (b-t)^{\alpha-1} p(t) \int_a^b (b-s)^{\alpha-1}(s-t)p(s)\mathrm{d}s\mathrm{d}t \left(g'(x)\right) \mathrm{d}x \\ & = \int_a^b H(x) g'(x) \mathrm{d}x, \end{aligned} \tag{14}$$

with

$$\begin{aligned} H(x) & = \frac{1}{2\Gamma^2(\alpha)} \int_a^x (b-t)^{\alpha-1} p(t) \int_a^b (b-s)^{\alpha-1}(s-t)p(s)\mathrm{d}s\mathrm{d}t \\ & = \frac{1}{2\Gamma^2(\alpha)} \left[\int_a^b s(b-s)^{\alpha-1} p(s)\mathrm{d}s \int_a^x (b-t)^{\alpha-1} p(t)\mathrm{d}t \right. \\ & \left. - \int_a^b (b-s)^{\alpha-1} p(s)\mathrm{d}s \int_a^x t(b-t)^{\alpha-1} p(t)\mathrm{d}t \right] \\ & = \frac{1}{2\Gamma(\alpha)} \left[\left(J_a^\alpha bp(b) \int_a^x (b-t)^{\alpha-1} p(t)\mathrm{d}t\right)\right. \\ & \left. - J_a^\alpha p(b) \int_a^x t(b-t)^{\alpha-1} p(t)\mathrm{d}t \right]. \end{aligned} \tag{15}$$

On the other hand, we have

$$
\begin{aligned}
&J_a^\alpha p(b) J_a^\alpha p\phi^2(b) - \left(J_a^\alpha p\phi(b)\right)^2 \\
&= \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \left[\phi(s)-\phi(t)\right]^2 \mathrm{d}s\mathrm{d}t \\
&= \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \\
&\times (s-t)^2 \left[\frac{\phi(s)-\phi(t)}{(s-t)}\right]^2 \mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{16}
$$

Hence,

$$
\begin{aligned}
&J_a^\alpha p(b) J_a^\alpha p\phi^2(b) - \left(J_a^\alpha p\phi(b)\right)^2 \\
&= \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t)(s-t)^2 \\
&\times \left[\frac{\int_t^s (\phi'(x))\,\mathrm{d}x}{(s-t)}\right]^2 \mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{17}
$$

Applying Cauchy-Schwarz inequality to the right hand side of (17), it yields that

$$
\begin{aligned}
&\tilde{T}(\phi,\phi,p) \\
&\le \frac{1}{2\Gamma^2(\alpha)} \left(\int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t)(s-t)^2 \mathrm{d}s\mathrm{d}t\right) \\
&\times \left[\frac{\left(\int_s^t 1\mathrm{d}x\right)^{\frac{1}{2}} \left(\int_t^s (\phi'(x))^2\,\mathrm{d}x\right)^{\frac{1}{2}}}{(s-t)}\right]^2 \\
&= \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t)(s-t) \\
&\times \left(\int_t^s \left(\phi'(x)\right)^2 \mathrm{d}x\right) \mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{18}
$$

Then, using (14) and (18), we get (9). □

REMARK 3.2. If we take $\alpha = 1$ in Theorem 3.1, we obtain a result similar to [1, Lemma 2.1] (this means that our results can be seen as a fractional equivalent version of the corresponding results in [1]).

Corollary 3.3. *Suppose that $\phi : [a,b] \to \mathbb{R}$ is an absolutely continuous function. If $(\phi')^2 \in L^1[a,b]$, then, for all $\alpha > 0$, we have*

$$\begin{aligned}&\frac{(b-a)^\alpha}{\Gamma(\alpha+1)} J_a^\alpha \phi^2(b) - (J_a^\alpha \phi(b))^2 \\ &\le \frac{1}{2\Gamma(\alpha)} \int_a^b \left[\left(J_a^\alpha b \int_a^x (b-t)^{\alpha-1} \mathrm{d}t\right) - \frac{(b-a)^\alpha}{\Gamma(\alpha+1)} \int_a^x t(b-t)^{\alpha-1} \mathrm{d}t\right] \\ &\qquad \times \left(\phi'(x)\right)^2 \mathrm{d}x.\end{aligned} \tag{19}$$

Proof. We take $p(x) = 1, x \in [a,b]$ in $H(x)$, in (10). So, we get the following expression for $H_1(x)$:

$$\begin{aligned}&H_1(x) \\ &= \frac{1}{2\Gamma(\alpha)} \left[\left(J_a^\alpha b \int_a^x (b-t)^{\alpha-1} \mathrm{d}t\right) - \frac{(b-a)^\alpha}{\Gamma(\alpha+1)} \int_a^x t(b-t)^{\alpha-1} \mathrm{d}t\right].\end{aligned} \tag{20}$$

Then, by Theorem 3.1, we deduce (19). □

Remark 3.4. If we take $\alpha = 1$ in Corollary 3.1, we obtain a result that is similar to [1, Corollary 2.2].

We also prove the following result.

Theorem 3.5. *Suppose that $f, g : [a,b] \to \mathbb{R}$ are two absolutely continuous functions on $[a,b]$ and $p : [a,b] \to \mathbb{R}^+$. If $(f')^2, (g')^2 \in L^1[a,b]$, then, for any $\alpha > 0$, we have*

$$\begin{aligned}&\left|J_a^\alpha p(b) J_a^\alpha pfg(b) - \left(J_a^\alpha pf(b)\right)\left(J_a^\alpha pg(b)\right)\right| \\ &\le \left(\int_a^b H(x) \left(f'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \left(\int_a^b H(x) \left(g'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}}.\end{aligned} \tag{21}$$

Proof. Using the weighted fractional Cauchy-Schwarz inequality for double integrals [7], we can write

$$\begin{aligned}&\left|J_a^\alpha p(b) J_a^\alpha pfg(b) - \left(J_a^\alpha pf(b)\right)\left(J_a^\alpha pg(b)\right)\right| \\ &\le \frac{1}{2\Gamma^2(\alpha)} \left(\int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \left(f(s)-f(t)\right)^2 \mathrm{d}s\mathrm{d}t\right)^{\frac{1}{2}} \\ &\qquad \times \left(\int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \left(g(s)-g(t)\right)^2 \mathrm{d}s\mathrm{d}t\right)^{\frac{1}{2}} \\ &= \left[J_a^\alpha p(b) J_a^\alpha pf^2(b) - \left(J_a^\alpha pf(b)\right)^2\right] \times \left[J_a^\alpha p(b) J_a^\alpha pg^2(b) - \left(J_a^\alpha pg(b)\right)^2\right].\end{aligned}$$

Since $(f')^2$ and $(g')^2 \in L^1[a,b]$, then, thanks to Theorem 3.1, we obtain the desired inequality. □

A no-weighted version for the above result can be given as follows.

Corollary 3.6. *Suppose that $f, g : [a, b] \to \mathbb{R}$ are two absolutely continuous functions. If $(f')^2, (g')^2 \in L^1[a, b]$, then, for any $\alpha > 0$, we have*

$$\begin{aligned}&\left|\frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)} J_a^{\alpha} fg(b) - J_a^{\alpha} f(b) J_a^{\alpha} g(b)\right| \\ &\quad\le \left(\int_a^b H_1(x)\left(f'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \left(\int_a^b H_1\left(g'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}},\end{aligned} \tag{22}$$

where H_1 is given by (20).

Proof. In fact, we have

$$\begin{aligned}&\left|\frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)} J_a^{\alpha} fg(b) - J_a^{\alpha} f(b) J_a^{\alpha} g(b)\right| \\ &\qquad\le \left(\int_a^b H_1(x)\left(f'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \left(\int_a^b H_1\left(g'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \\ &= \left(\frac{1}{2\Gamma(\alpha)}\int_a^b \left[\left(J_a^{\alpha} b \int_a^x (b-t)^{\alpha-1}\mathrm{d}t\right) - \frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)}\int_a^x t(b-t)^{\alpha-1}\mathrm{d}t\right]\right. \\ &\quad \left.\times \left(f'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \left(\frac{1}{2}\int_a^b \left[\left(J_a^{\alpha} b \int_a^x (b-t)^{\alpha-1}\mathrm{d}t\right) - \frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)}\right.\right. \\ &\qquad \left.\left.\times \int_a^x t(b-t)^{\alpha-1}\mathrm{d}t\right] \left(g'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \\ &= \frac{1}{2\Gamma(\alpha)}\left(\int_a^b \left[\left(J_a^{\alpha} b \int_a^x (b-t)^{\alpha-1}\mathrm{d}t\right) - \frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)}\int_a^x t(b-t)^{\alpha-1}\mathrm{d}t\right]\right. \\ &\quad \left.\times \left(f'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}} \left(\int_a^b \left[\left(J_a^{\alpha} b \int_a^x (b-t)^{\alpha-1}\mathrm{d}t\right) - \frac{(b-a)^{\alpha}}{\Gamma(\alpha+1)}\right.\right. \\ &\qquad\qquad \left.\left.\times \int_a^x t(b-t)^{\alpha-1}\mathrm{d}t\right] \left(g'(x)\right)^2 \mathrm{d}x\right)^{\frac{1}{2}}.\end{aligned}$$

□

Remark 3.7. Taking $\alpha = 1$ in Corollary 3.2, we get [1, Corollary 2.4].

In the case of a nondecreasing function, we prove the following result.

Theorem 3.8. *Let g be a nondecreasing function on $[a, b]$, f be an absolutely continuous function on $[a, b]$ and suppose that p is a positive and integrable function on $[a, b]$. If $f' \in L^{\infty}[a, b]$, then, for any $\alpha > 0$, we have*

$$|J_a^{\alpha} p(b) J_a^{\alpha} pfg(b) - J_a^{\alpha} pf(b) J_a^{\alpha} pg(b)| \le ||f'||_{\infty} \int_a^b H(x) g'(x)\mathrm{d}x. \tag{23}$$

Proof. Since

$$
\begin{aligned}
&|J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b)| \\
&= \frac{1}{2\Gamma^2(\alpha)} \left| \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \right. \\
&\qquad \left. \times \left[(f(s)-f(t)) \, (g(s)-g(t)) \right] \mathrm{d}s\mathrm{d}t \right|,
\end{aligned} \tag{24}
$$

we can write

$$
\begin{aligned}
&|J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b)| \\
&\leq \frac{1}{2\Gamma^2(\alpha)} \int_a^b \int_a^b (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t) \\
&\qquad \times \left| \frac{(f(s)-f(t))}{s-t} \right| |(s-t) \, (g(s)-g(t))| \, \mathrm{d}s\mathrm{d}t.
\end{aligned} \tag{25}
$$

By $f' \in L^\infty [a,b]$, we have that

$$
\begin{aligned}
&|J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b)| \\
&\leq \frac{||f'||_\infty}{2\Gamma^2(\alpha)} \int_a^b \int_a^x (b-s)^{\alpha-1}(b-t)^{\alpha-1} p(s)p(t)(s-t) \\
&\qquad \times \left(\int_a^b g'(x)\mathrm{d}x \right) \mathrm{d}s\mathrm{d}t \\
&= \frac{||f'||_\infty}{2\Gamma(\alpha)} \left[\left(J_a^\alpha bp(b) \int_a^x (b-t)^{\alpha-1} p(t)\mathrm{d}t \right) - J_a^\alpha p(b) \right. \\
&\qquad \left. \times \int_a^x t(b-t)^{\alpha-1} p(t)\mathrm{d}t \right] \left(\int_a^b g'(x)\mathrm{d}x \right).
\end{aligned} \tag{26}
$$

Thanks to (10), we get (23). □

Remark 3.9. Taking $\alpha = 1$ in Theorem 3.3, we obtain [1, Theorem 2.5].

Corollary 3.10. *Let g be a nondecreasing function on $[a,b]$ and f be an absolutely continuous function. If $f' \in L^\infty[a,b]$, then for any $\alpha > 0$, we have*

$$
\left| \frac{(b-a)^\alpha}{\Gamma(\alpha+1)} J_a^\alpha fg(b) - J_a^\alpha f(b) J_a^\alpha g(b) \right| \leq ||f'||_\infty \left(\int_a^b H_1(x) g'(x)\mathrm{d}x \right), \tag{27}
$$

where H_1 is given by (20).

Remark 3.11. Taking $\alpha = 1$ in Corollary 3.3, we obtain [3, Theorem 2.6].

The main result corresponding to the case of two monotonic functions is given by the following theorem.

Theorem 3.12. *Let $f, g : [a,b] \to \mathbb{R}$ be two absolutely monotonic functions on $[a,b]$ such that g be non decreasing on $[a,b]$. If $f', g' \in L^\infty[a,b]$, then, for any positive function p defined on $[a,b]$, we have*

$$
|J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b)| \leq ||f'||_\infty ||g'||_\infty \int_a^b H(x)\mathrm{d}x. \tag{28}
$$

Proof. It is easy to see that

$$
\begin{aligned}
&|J_a^\alpha p(b) J_a^\alpha pfg(b) - J_a^\alpha pf(b) J_a^\alpha pg(b)| \\
&\leq ||f'||_\infty \int_a^b H(x) g'(x) \mathrm{d}x \\
&\leq ||f'||_\infty ||g'||_\infty \int_a^b H(x) \mathrm{d}x.
\end{aligned} \tag{29}
$$

□

Remark 3.13. Taking $\alpha = 1$ in theorem 3.4, we obtain [1, Theorem 2.7].

To finish, we present the following result.

Corollary 3.14. *let $f, g : [a,b] \to \mathbb{R}$ be two absolutely monotonic functions on $[a,b]$ and g be non decreasing on $[a,b]$. If $f', g' \in L^\infty[a,b]$, then, for $\alpha > 0$, we have the inequality*

$$
\left| \frac{(b-a)^\alpha}{\Gamma(\alpha+1)} J_a^\alpha fg(b) - J_a^\alpha f(b) J_a^\alpha g(b) \right| \leq ||f'||_\infty ||g'||_\infty \int_a^b H_1(x) \mathrm{d}x. \tag{30}
$$

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