MATHEMATICA, 59 (82), N° 1–2, 2017, pp. 102–110

FUNDAMENTAL STABILITIES OF GENERALIZED COMPOSITE FUNCTIONAL EQUATIONS IN NON-ARCHIMEDEAN SPACES

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Abstract. In this paper, we introduce a new generalized composite functional equation of the form

$$f\left(kf(x_1)-\sum_{i=2}^{k+1}f(x_i)\right)+kf\left(x_1\right)+\sum_{i=2}^{k+1}f(x_i)=\sum_{i=2}^{k+1}f(x_1+x_i)+\sum_{i=2}^{k+1}f(x_1-x_i),$$

for any real $k\in\mathbb{R}^+\setminus\{0\}$, and prove its fundamental stabilities in non-Archimedean normed spaces.

MSC 2010. 39B55, 39B52, 39B82.

Key words. Composite functional equation, non-Archimedean space, Hyers-Ulam stability.

1. INTRODUCTION

The stability problem of functional equations originates from the fundamental question: When is it true that a mathematical object, satisfying approximately a certain property, has to be close to an object satisfying exactly that property? The first stability problem concerning group homomorphisms was raised by Ulam [19] in 1940. The first partial solution to Ulam's question was given by Hyers [7]. In 1978, Rassias [14] proved a generalization of Hyers' theorem for additive maps. More exactly, he proved the following theorem.

THEOREM 1.1. *If a function $f:E\to E'$ between the Banach spaces E, E' satisfies the inequality*

$$\|f(x+y)-f(x)-f(y)\|\le\theta\left(\|x\|^p+\|y\|^p\right) \tag{1}$$

for some $\theta\ge0$, $0\le p<1$ and for all $x,y\in E$, then there exists a unique additive function $a:E\to E'$ such that

$$\|f(x)-a(x)\|\le\frac{2\theta}{2-2^p}\|x\|^p, \tag{2}$$

for all $x\in E$. Moreover, if $f(tx)$ is continuous in t, for each fixed $x\in E$, then a is linear.

The authors would like to thank the anonymous reviewer and the editor for their very helpful comments and suggestions.

The result of Th. M. Rassias has provided a significant influence during the last three decades in the development of a generalization of the Hyers-Ulam stability concept. This new concept is known as the Hyers-Ulam-Rassias stability of functional equations.

In the period 1982-1989, J. M. Rassias [12, 13] replaced the sum in the right hand side of the equation (1) by the product of powers of norms. This stability is called the Ulam-Găvruta-Rassias stability involving a product of different powers of norms.

Recently, J. M. Rassias replaced in [15] the sum in the right hand side of the equation (1) by the mixed product-sum of powers of norms. The study of the stability of the functional equation involving the mixed product-sum of powers of norms is known as the Ulam-J. Rassias stability.

During the last decades, several stability problems of functional equations have been investigated (see [4, 5, 8, 9, 11, 16, 17]).

Definition 1.2. A *non-Archimedean field* is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ such that

(i) $|r| = 0$ if and only if $r = 0$,
(ii) $|rs| = |r||s|$,
(iii) $|r+s| \leq \max\{|r|, |s|\}$,

for all $r, s \in \mathbb{K}$. Clearly $|1| = |-1| = 1$ and $|n| \leq 1$ for all $n \in \mathbb{N}$. We always assume, in addition, that $|\cdot|$ is non-trivial, i.e., there exists an $a_0 \in \mathbb{K}$ such that $|a_0| \neq 0, 1$.

Definition 1.3. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a *non-Archimedean norm* (*valuation*), if it satisfies the following conditions:

(i) $\|x\| = 0$ if and only if $x = 0$,
(ii) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}, x \in X$,
(iii) the strong triangle inequality (ultrametric), namely,

$$\|x+y\| \leq \max\{\|x\|, \|y\|\}, \ x, y \in X.$$

Due to the fact that

$$\|x_n - x_m\| \leq \max\{\|x_{j+1} - x_j\| : m \leq j \leq n-1\}, \ n > m,$$

a sequence $\{x_n\}$ is Cauchy if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean space. By a *complete non-Archimedean space*, we mean one in which every Cauchy sequence is convergent.

It is generally accepted that non-Archimedean spaces are more fundamental than the standard complete normed spaces to investigate Ulam's stability of any equation. Therefore, the benefit of such spaces is greater than those of others.

In 2011, H.Azadi Kenary [1] investigated the generalized Hyers-Ulam stability of the following Cauchy-Jensen type functional equation

$$Q\left(\frac{x+y}{2}+z\right)+Q\left(\frac{x+z}{2}+y\right)+Q\left(\frac{z+y}{2}+x\right)$$
$$=2[Q(x)+Q(y)+Q(z)]$$

in non-Archimedean spaces. Furthermore, in 2013, H.Azadi Kenary [2] proved the generalized Hyers-Ulam (or Hyers-Ulam-Rassias) stability of the following composite functional equation

$$f\left(f(x)-f(y)\right)+f(x)+f(y)=f(x+y)+f(x-y)$$

in various normed spaces. The above equation is satisfied by the general additive map $f(x)=cx$, where c is a real constant.

In 2015, Ravi and Ponmanaselvan[18] investigated a composite type functional equation of the form

$$f\left(xf(y)-yf(x)\right)=f(x)-f(y)+x-y$$

on an abelian group.

Very recently, A. Bodaghi, P. Narasimman, J. M. Rassias and K. Ravi introduced in [6] a new generalized reciprocal functional equation and studied its Hyers-Ulam-Rassias stability. Also, they provided counterexamples for some cases, such as the Ulam-Găvruta-Rassias stability and the Hyers-Ulam-Rassias stability in non-Archimedean fields.

We now introduce a new generalized composite functional equation of the form

$$f\left(kf(x_1)-\sum_{i=2}^{k+1}f(x_i)\right)+kf\left(x_1\right)+\sum_{i=2}^{k+1}f(x_i)= \sum_{i=2}^{k+1}f(x_1+x_i)+\sum_{i=2}^{k+1}f(x_1-x_i), \tag{3}$$

for any real $k\in\mathbb{R}^+\setminus\{0\}$, which is satisfied by the map $f(x)=cx$, c being a real constant. We study in the next section the Hyers-Ulam-Rassias stability, the Ulam-Găvruta-Rassias stability and the Ulam-J. Rassias stability of the functional equation (3) in non-Archimedean normed spaces.

2. HYERS-ULAM-RASSIAS STABILITY OF (3)

In this section, we prove the Hyers-Ulam-Rassias stability, the Ulam-Găvruta-Rassias stability and the Ulam-J.Rassias stability of the functional equation (3) in non-Archimedean normed spaces.

THEOREM 2.1. *Let $\varsigma : G\times G\to[0,\infty)$ be a function such that*

$$\lim_{k\to\infty}\frac{|2|^k}{|k|}\varsigma\left(\frac{x_1}{2^k},...,\frac{x_k}{2^k},\frac{x_{k+1}}{2^k}\right)=0, \tag{4}$$

for all $x_1, x_2, ..., x_k, x_{k+1} \in G$. *Suppose that, for any* $x \in G$, *the limit*

$$\Psi(x) = \lim_{k\to\infty} \max\left\{|2|^{n+1}\varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, ..., \frac{x}{2^{n+1}}\right) :\ 0 \le n < k\right\} \tag{5}$$

exists, and that $f : G \to X$ *is a map satisfying*

$$\begin{aligned}\Big\| f\left(kf(x_1) - \sum_{i=2}^{k+1} f(x_i)\right) + kf(x_1) + \sum_{i=2}^{k+1} f(x_i) \\ - \sum_{i=2}^{k+1} f(x_1 + x_i) - \sum_{i=2}^{k+1} f(x_1 - x_i)\Big\| \le \varsigma(x_1, x_1, ..., x_k, x_{k+1}),\end{aligned} \tag{6}$$

for all $x_1, x_2, ..., x_k, x_{k+1} \in G$. *Then, for all* $x \in G$, *the limit* $T(x) := \lim\limits_{k\to\infty} 2^k f\left(\frac{x}{2^k}\right)$ *exists and satisfies the inequality*

$$\|f(x) - T(x)\| \le \frac{1}{|2k|}\Psi(x). \tag{7}$$

Moreover, if

$$\lim_{j\to\infty}\lim_{k\to\infty} \max\left\{|2|^{n+1}\varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, ..., \frac{x}{2^{n+1}}\right) :\ j \le n < k + j\right\} = 0, \tag{8}$$

then T *is the unique additive map satisfying* (7).

Proof. Putting $x_1, x_2, \ldots, x_k, x_{k+1} = x$ in (6), we have

$$\|2f(x) - f(2x)\| \le \frac{1}{|k|}\varsigma(x, x, ..., x)_{k+1 \text{ times}}, \tag{9}$$

for all $x \in G$. Taking x to be $\frac{x}{2}$ in (9), we obtain

$$\left\|2f(\frac{x}{2}) - f(x)\right\| \le \frac{1}{|k|}\varsigma\left(\frac{x}{2}, \frac{x}{2}, ..., \frac{x}{2}\right)_{k+1 \text{ times}}, \tag{10}$$

for all $x \in G$. Taking x to be $\frac{x}{2^k}$ in (10), we obtain

$$\left\|2^{k+1}f(\frac{x}{2^{k+1}}) - 2^k f(\frac{x}{2^k})\right\| \le \frac{|2|^k}{|k|}\varsigma\left(\frac{x}{2^{k+1}}, \frac{x}{2^{k+1}}, ..., \frac{x}{2^{k+1}}\right)_{k+1 \text{ times}}. \tag{11}$$

Thus, it follows from (4) and (11) that the sequence $2^k f(\frac{x}{2^k})_{k\ge1}$ is a Cauchy sequence. Since X is complete, it follows that $2^k f(\frac{x}{2^k})_{k\ge1}$ is convergent. Set $T(x) = \lim_{n\to\infty} 2^k f(\frac{x}{2^k})$. Using induction, one can show that

$$\left\|2^k f(\frac{x}{2^k}) - f(x)\right\| = \frac{\max\left\{|2|^{n+1}\varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, ..., \frac{x}{2^{n+1}}\right) :\ 0 \le n < k\right\}}{|k||2|}, \tag{12}$$

for all $k \geq 1$ and $x \in G$. By taking $k \to \infty$ in (12) and using (5), one obtains (7). By (4) and (6), we get

$$\begin{aligned}
&\left\| T\left(kT(x_1) - \sum_{i=2}^{k+1} T(x_i)\right) + kT\left(x_1\right) + \sum_{i=2}^{k+1} T(x_i) \right.\\
&\qquad\qquad \left. - \sum_{i=2}^{k+1} T(x_1 + x_i) - \sum_{i=2}^{k+1} T(x_1 - x_i) \right\| \\
&= \lim_{k\to\infty} \left\| 2^k \left[f\left(kf\left(\frac{x_1}{2^k}\right) - \sum_{i=2}^{k+1} f\left(\frac{x_i}{2^k}\right)\right) + kf\left(\frac{x_1}{2^k}\right) + \sum_{i=2}^{k+1} f\left(\frac{x_i}{2^k}\right) \right.\right.\\
&\qquad\qquad \left.\left. - \sum_{i=2}^{k+1} f\left(\frac{x_1 + x_i}{2^k}\right) - \sum_{i=2}^{k+1} f\left(\frac{x_1 - x_i}{2^k}\right) \right] \right\| \\
&\qquad\qquad\qquad\qquad \leq \lim_{k\to\infty} |2|^k \varsigma\left(\frac{x_1}{2^k}, \frac{x_2}{2^k}, ..., \frac{x_k}{2^k}, \frac{x_{k+1}}{2^k}\right) = 0,
\end{aligned}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in G$. Therefore, the map $T : G \to X$ satisfies (3). To prove the uniqueness property of T, let S be another map satisfying (7). Then we have

$$\begin{aligned}
\|T(x) - S(x)\| &= \lim_{j\to\infty} |2|^j \left\| T\left(\frac{x}{2^j}\right) - S\left(\frac{x}{2^j}\right) \right\| \\
&\leq \lim_{j\to\infty} |2|^j \max\left\{ \left\| T\left(\frac{x}{2^j}\right) - f\left(\frac{x}{2^j}\right) \right\|, \left\| f\left(\frac{x}{2^j}\right) - S\left(\frac{x}{2^j}\right) \right\| \right\} \\
&\leq \lim_{j\to\infty} \lim_{k\to\infty} \frac{1}{|2k|} \max\Big\{ |2|^{n+1} \varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, ..., \frac{x}{2^{n+1}}\right) : \\
&\qquad\qquad j \leq n < k + j \Big\} \\
&= 0,
\end{aligned}$$

for all $x \in G$. Therefore, $T = S$. This completes the proof. □

Corollary 2.2. *Let $\xi : [0, \infty) \to [0, \infty)$ be a function satisfying*

$$\xi\left(\frac{t}{|2|}\right) \leq \xi\left(\frac{t}{|2|}\right) \xi(t), \;\; \xi\left(\frac{1}{|2|}\right) < \frac{1}{|2|}, \tag{13}$$

for all $t \geq 0$. Let $\iota > 0$ and let $f : G \to X$ be a map such that

$$\begin{aligned}
&\left\| f\left(kf(x_1) - \sum_{i=2}^{k+1} f(x_i)\right) + kf\left(x_1\right) + \sum_{i=2}^{k+1} f(x_i) - \sum_{i=2}^{k+1} f(x_1 + x_i) \right.\\
&\qquad \left. - \sum_{i=2}^{k+1} f(x_1 - x_i) \right\| \leq \iota\big(\xi(|x_1|) + \xi(|x_2|) + ... + \xi(|x_{k+1}|)\big),
\end{aligned} \tag{14}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in G$. Then there exists a unique additive map $T : G \to X$ such that

$$\|f(x) - T(x)\| \leq \frac{(k+1)\ \iota\ \xi(|x|)}{|2k|}. \tag{15}$$

Proof. If we define $\varsigma : G \times G \to [0, \infty)$ by

$$\varsigma(x_1, x_2, \ldots, x_{k+1}) = \iota\big(\xi(|x_1|) + \xi(|x_2|) + \ldots + \xi(|x_{k+1}|)\big),$$

then we have that

$$\begin{aligned}&\lim_{k\to\infty} \frac{|2|^k}{|k|} \varsigma\left(\frac{x_1}{2^k}, \ldots, \frac{x_k}{2^k}, \frac{x_{k+1}}{2^k}\right)\\ &\quad\leq \lim_{k\to\infty} \frac{1}{|k|}\left(|2|\xi\left(\frac{1}{|2|}\right)\right)^k \left[\iota\big(\xi(|x_1|) + \xi(|x_2|) + \ldots + \xi(|x_{k+1}|)\big)\right] = 0\end{aligned}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in G$. On the other hand, for all $x \in G$,

$$\begin{aligned}\Psi(x) &= \lim_{k\to\infty} \max\left\{|2|^{n+1}\varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, \ldots, \frac{x}{2^{n+1}}\right) :\ 0 \leq n < k\right\}\\ &= |2|\varsigma\left(\frac{x}{2}, \frac{x}{2}, \ldots, \frac{x}{2}\right)\\ &= \iota(k+1)\xi(|x|)\end{aligned}$$

exists. Also, we have

$$\begin{aligned}&\lim_{j\to\infty}\lim_{k\to\infty} \max\left\{|2|^{n+1}\varsigma\left(\frac{x}{2^{n+1}}, \frac{x}{2^{n+1}}, \ldots, \frac{x}{2^{n+1}}\right) :\ j \leq n < k+j\right\}\\ &\quad= \lim_{j\to\infty} |2|^{j+1}\varsigma\left(\frac{x}{2^{j+1}}, \frac{x}{2^{j+1}}, \ldots, \frac{x}{2^{j+1}}\right) = 0.\end{aligned}$$

Thus, applying Theorem 2.1, we get the desired result. □

THEOREM 2.3. *Let $\varsigma : G \times G \to [0, \infty)$ be a function such that*

$$\lim_{k\to\infty} \frac{|2|^k}{|k|}\varsigma\big(2^k x_1, 2^k x_2, \ldots, 2^k x_k, 2^k x_{k+1}\big) = 0, \tag{16}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in G$. Suppose that, for every $x \in G$, the limit

$$\Psi(x) = \lim_{k\to\infty} \max\left\{\frac{\varsigma\big(2^n x, , \ldots, 2^n x\big)}{|2|^n} :\ 0 \leq n < k\right\} \tag{17}$$

exists and that $f : G \to X$ is a map satisfying (6)*. Then the limit $T(x) := \lim_{k\to\infty} \frac{f(2^k x)}{2^k}$ exists, for all $x \in G$, and satisfies the inequality*

$$\|f(x) - T(x)\| \leq \frac{1}{|2k|}\Psi(x). \tag{18}$$

Moreover, if

$$\lim_{j\to\infty}\lim_{k\to\infty} \max\left\{\frac{\varsigma\big(2^n x, 2^n x, \ldots, 2^n x\big)}{|2|^n} :\ j \leq n < k+j\right\} = 0, \tag{19}$$

then T is the unique additive map satisfying (18).

Proof. By (9), we have

$$\left\| f(x) - \frac{f(2x)}{2} \right\| \leq \frac{1}{|2k|} \varsigma(x, x, \ldots, x)_{k+1 \ times}, \tag{20}$$

for all $x \in G$. Taking x to be $2^k x$ in (20), we obtain

$$\left\| \frac{f(2^k x)}{2^k} - \frac{f(2^{k+1} x)}{2^{k+1}} \right\| \leq \frac{1}{|k||2|^{k+1}} \varsigma(2^k x, 2^k x, \ldots, 2^k x)_{k+1 \text{ times}}. \tag{21}$$

Thus, it follows from (16) and (21) that the sequence $\left\{ \frac{2^k x}{2^k} \right\}_{k \geq 1}$ is convergent. Set $T(x) = \lim_{k\to\infty} \frac{f(2^k x)}{2^k}$. On the other hand, it follows from (21) that

$$\begin{aligned}
\left\| \frac{f(2^p x)}{2^p} - \frac{f(2^q x)}{2^q} \right\| &= \left\| \sum_{n=p}^{q-1} \frac{f(2^k x)}{2^k} - \frac{f(2^{k+1} x)}{2^{k+1}} \right\| \\
&\leq \max \left\{ \left\| \frac{f(2^k x)}{2^k} - \frac{f(2^{k+1} x)}{2^{k+1}} \right\| : \ p \leq n < q \right\} \\
&\leq \frac{1}{|2k|} \max \left\{ \frac{\varsigma\big(2^n x, 2^n x, \ldots, 2^n x\big)}{|2|^n} : \ p \leq n < q \right\},
\end{aligned}$$

for all $x \in G$ and all integers $p, q \geq 0$ with $q > p \geq 0$. Letting $p = 0$, taking $q \to \infty$ in the last inequality and using (17), we obtain (18).

The rest of the proof is similar to the proof of Theorem 2.1. This completes the proof. $\square$

COROLLARY 2.4. *Let $\xi : [0, \infty) \to [0, \infty)$ be a function satisfying*

$$\xi\left(|2|t\right) \leq \xi\left(|2|\right)\xi(t), \ \ \xi\left(|2|\right) < |2|, \tag{22}$$

for all $t \geq 0$. Let $\iota > 0$ and let $f : G \to X$ be a map satisfying (14). *Then there exists a unique additive map $T : G \to X$ such that*

$$\|f(x) - T(x)\| \leq \frac{(k+1)\ \iota\ \xi(|x|)}{|2k|}. \tag{23}$$

Proof. If we define $\varsigma : G \times G \to [0, \infty)$ by

$$\varsigma(x_1, x_2, \ldots, x_{k+1}) = \iota\big(\xi(|x_1|) + \xi(|x_2|) + \ldots + \xi(|x_{k+1}|)\big),$$

we get that

$$\begin{aligned}
&\lim_{k\to\infty} \frac{\varsigma\big(2^k x_1, 2^k x_2, \ldots, 2^k x_k, 2^k x_{k+1}\big)}{|2|^k |k|} \\
&\quad \leq \lim_{k\to\infty} \frac{1}{|k|} \left(\frac{1}{|2|} \xi\left(|2|\right) \right)^k \big[\iota\big(\xi(|x_1|) + \xi(|x_2|) + \ldots + \xi(|x_{k+1}|)\big)\big] = 0,
\end{aligned}$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in G$. On the other hand, for all $x \in G$,

$$\Psi(x) = \lim_{k\to\infty} \max\left\{ \frac{\varsigma\big(2^n x, 2^n x, \ldots, 2^n x\big)}{|2|^n} \,:\, 0 \le n < k \right\}$$
$$= \frac{1}{|2|}\left(\iota\big(\xi(|2x|) + \ldots + \xi(|2x|)\big)\right) = \iota(k+1)\xi(|x|)$$

exists. Also, we have

$$\lim_{j\to\infty}\lim_{k\to\infty} \max\left\{ \frac{\varsigma\big(2^n x, 2^n x, \ldots, 2^n x\big)}{|2|^n} \,:\, j \le n < k+j \right\}$$
$$= \lim_{j\to\infty} \frac{\varsigma\big(2^j x, 2^j x, \ldots, 2^j x\big)}{|2|^j} = 0.$$

Thus, applying Theorem 2.3, we get the desired result. □

3. CONCLUSION

In this paper, we have introduced a new generalized composite functional equation and we have studied its fundamental stabilities in non-Archimedean normed spaces.

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Received April 25, 2017
Accepted July 13, 2017

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