MATHEMATICA, Tome 49(72), N° 1, 2007, pp. 55–61

THE FEKETE-SZEGÖ INEQUALITY FOR A SUBCLASS OF ANALYTIC FUNCTIONS INVOLVING HADAMARD PRODUCT

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Abstract. For $0 < \alpha \le 1$, $0 \le \beta \le \lambda \le 1$, $0 \le \delta < 1$, $0 \le \nu < 1$ and $\rho > 0$, let $\Re(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$ be the class of analytic functions defined in the open unit disk E by

$$\left|\arg\left(\frac{\lambda\beta z^3 f'''(z) + (2\lambda\beta + \lambda - \beta)z^2 f''(z) + zf'(z)}{\lambda\beta z^2 g''(z) + (\lambda - \beta)zg'(z) + (1 - \lambda + \beta)f(z)} - \delta\right)\right| < \frac{\pi\alpha}{2} \qquad (z \in E),$$

where $g(z) = z + b_2z^2 + b_3z^3 + \ldots$ is analytic function on E and satisfies

$$\left|\arg\left(\frac{g(z) * \Phi(z)}{g(z) * \Psi(z)} - \nu\right)\right| < \frac{\pi\rho}{2} \qquad (z \in E),$$

for some $\Phi(z) = z + \sum_{n=2}^{\infty} \Upsilon_n z^n$ and $\Psi(z) = z + \sum_{n=2}^{\infty} \gamma_n z^n$ analytic in E such that $g(z) * \Psi(z) \ne 0, \Upsilon_n \ge 0, \gamma_n \ge 0$ and $\Upsilon_n > \gamma_n (n \ge 2)$. For $f \in \Re(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$ and given by $f(z) = z + a_2z^2 + a_3z^3 + \cdots$, a sharp upper bound is obtained for functional $\left|a_3 - \mu a_2^2\right|$ when $\mu \ge 1$.

MSC 2000. 30C45.
Key words. Analytic functions, Univalent functions, Starlike functions, Convex functions, Fekete-Szegö problem.

1. INTRODUCTION

Let A denote the family of the functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disk $E = \{z : z \in \mathrm{C} \text{ and } |z| < 1\}$. Let S denote the class of functions which are univalent in E.

Let the function $f(z)$ be defined by (1). Also let the function $g(z)$ be defined by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n. \tag{2}$$

Then the Hadamard product (or convolution) of the functions $f(z)$ and $g(z)$ is defined by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n. \tag{3}$$

Fekete and Szegö [9] obtained sharp upper bound for functional $\left|a_3 - \mu a_2^2\right|$ when μ is real. For various subclasses of S, namely the class of starlike functions, the class of convex functions, and the class of close-to-convex functions denoted by S^*, C and K , respectively, sharp upper bound for functional $\left|a_3 - \mu a_2^2\right|$ has been investigated by many different authors including ([1]–[7], [10]–[12], [14]–[18], [21]-[22]). Obviously, the classical ones are from the work done by Fekete and Szegö [9], and Keogh and Merkes [14]. In the present paper we obtain sharp upper bounds for functional $\left|a_3 - \mu a_2^2\right|$ when f belonging to the class of functions defined as follows:

Definition 1. Let $0 < \alpha \leq 1,\ 0 \leq \beta \leq \lambda \leq 1,\ 0 \leq \delta < 1,\ 0 \leq \nu < 1$ and $\rho > 0$, and let $f \in A$. Then $f \in \Re(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$ if and only if

(4)
$$\left|\arg\left(\frac{\lambda\beta z^3 f'''(z) + (2\lambda\beta + \lambda - \beta)z^2 f''(z) + zf'(z)}{\lambda\beta z^2 g''(z) + (\lambda - \beta)zg'(z) + (1 - \lambda + \beta)f(z)} - \delta\right)\right| < \frac{\pi\alpha}{2} \quad (z \in E)$$

with $g \in A$ and satisfies

(5)
$$\left|\arg\left(\frac{g(z) * \Phi(z)}{g(z) * \Psi(z)} - \nu\right)\right| < \frac{\pi\rho}{2} \qquad (z \in E),$$

where $\Phi(z) = z + \sum_{n=2}^{\infty} \Upsilon_n z^n$ and $\Psi(z) = z + \sum_{n=2}^{\infty} \gamma_n z^n$ analytic in the open unit disk E such that $g(z) * \Psi(z) \neq 0, \Upsilon_n \geq 0, \gamma_n \geq 0$ and $\Upsilon_n > \gamma_n$ $(n \geq 2)$.

Note that $\Re(\frac{z}{(1-z)^2}, \frac{z}{1-z}; 0, 0, 1, 0, 0, \rho) = K(\rho)$ is the class of close-to-convex functions defined in [3] and $\Re(\frac{z}{(1-z)^2}, \frac{z}{1-z}; 0, 0, 1, 0, 0, 1) = K(1)$ is the class of normalized close-to-convex functions defined by Kaplan [13]. Whereas,

$$\Re(\frac{z}{(1-z)^2}, \frac{z}{1-z}; 0, 0, \alpha, 0, 0, \rho) = K(\alpha, \rho)$$

is the class of normalized close-to-convex functions defined in [6].

2. PRINCIPAL RESULTS

In order to derive our principal results, we have to recall here the following lemma [20].

Lemma 1. *Let $h \in P$ i.e., h be analytic in the open unit disk E and be given by*

$$h(z) = 1 + c_1 z + c_2 z^2 + \cdots ,$$

and $\mathrm{Re} h(z) > 0$ *for* $z \in E$, *then*

(6)
$$\left|c_2 - \frac{c_1^2}{2}\right| \leq 2 - \frac{|c_1|^2}{2}.$$

Making use of the above Lemma, we get the following theorem.

Theorem 1. *Let $f(z)$ be given by* (1). *If* $f(z) \in \Re(\Phi, \Psi; \lambda, \beta, \alpha, \delta, \nu, \rho)$; $0 < \alpha \leq 1,\ 0 \leq \beta \leq \lambda \leq 1,\ 0 \leq \delta < 1,\ 0 \leq \nu < 1,\ \rho \geq 1$ *and* $3\mu\sigma(1 + \delta) \geq$

$2\eta(\eta+2\gamma_2)$, *where* $\eta=\Upsilon_2-\gamma_2(1+\nu)$, $\sigma=\Upsilon_3-\gamma_3(1+\nu)$ *and* $\mu\geq 0$, *then we have the sharp inequality*

$$\left|a_3-\mu a_2^2\right| \leq \frac{\rho^2(1+\delta)}{3\sigma\eta^2}\{3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)\}$$

$$+\alpha\left\{\frac{3\mu[2(3\lambda\beta+\lambda-\beta)+1]\{\alpha\eta+2\rho[(2\lambda\beta+\lambda-\beta)+1](1+\delta)\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}\right.$$

$$\left.-\frac{2[(2\lambda\beta+\lambda-\beta)+1]^2\{\alpha\eta+2\rho[(2\lambda\beta+\lambda-\beta)+1]\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}\right\} \tag{7}$$

Proof. Let $f(z)\in\Re(\Phi,\Psi;\lambda,\beta,\alpha,\delta,\nu,\rho)$. It follows from (4) that

$$\lambda\beta z^3 f'''(z)+(2\lambda\beta+\lambda-\beta)z^2 f''(z)+zf'(z)$$

$$=[\lambda\beta z^2 g''(z)+(\lambda-\beta)zg'(z)+(1-\lambda+\beta)f(z)][q^\alpha(z)+\delta] \tag{8}$$

for $z\in E$, with $q\in P$ given by $q(z)=1+q_1z+q_2z^2+\cdots$. Equating coefficients, we get

$$2[(2\lambda\beta+\lambda-\beta)+1]a_2=[(2\lambda\beta+\lambda-\beta)+1](1+\delta)b_2+\alpha q_1 \tag{9}$$

and

$$3[2(3\lambda\beta+\lambda-\beta)+1]a_3=\alpha q_2+\alpha q_1[(2\lambda\beta+\lambda-\beta)+1]b_2$$

$$+\frac{\alpha(\alpha-1)}{2}q_1^2+(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1]b_3. \tag{10}$$

Also, it follows from (5) that

$$g(z)*\Phi(z)=[g(z)*\Psi(z)][p^\rho(z)+\nu], \tag{11}$$

where $p\in P$ with $p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots$ for $z\in E$. Thus, equating coefficients, we obtain

$$\eta b_2=\rho p_1 \tag{12}$$

and

$$\sigma b_3=\rho\left(p_2+\frac{\rho(\eta+2\gamma_2)-\eta}{2\eta}p_1^2\right). \tag{13}$$

From (9), (10), (12) and (13) we have

$$a_3-\mu a_2^2=\frac{\alpha}{3[2(3\lambda\beta+\lambda-\beta)+1]}(q_2-\frac{q_1^2}{2})+\frac{\rho(1+\delta)}{3\sigma}\left(p_2-\frac{p_1^2}{2}\right)$$

$$+\frac{\alpha^2\{2[(2\lambda\beta+\lambda-\beta)+1]^2-3\mu[2(3\lambda\beta+\lambda-\beta)+1]\}}{12[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}q_1^2$$

$$+\frac{\rho^2(1+\delta)[2\eta(\eta+2\gamma_2)-3\mu\sigma(1+\delta)]}{12\sigma\eta^2}p_1^2$$

$$+\frac{\alpha\rho\{2[(2\lambda\beta+\lambda-\beta)+1]^2-3\mu(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1]\}}{6\eta[(2\lambda\beta+\lambda-\beta)+1][2(3\lambda\beta+\lambda-\beta)+1]}p_1q_1. \tag{14}$$

Assume that $a_3-\mu a_2^2$ positive. Thus we now estimate $Re(a_3-\mu a_2^2)$ by applying the same technique done by London [17]. And so from (14) and by using Lemma 1 and letting $p_1 = 2re^{i\theta},\ q_1 = 2Re^{i\phi},\ 0 \le r \le 1,\ 0 \le R \le 1,\ 0 \le \theta \le 2\pi$ and $0 \le \phi \le 2\pi$, we obtain

$$Re(a_3 - \mu a_2^2) = \frac{\alpha}{3[2(3\lambda\beta+\lambda-\beta)+1]} Re(q_2 - \frac{q_1^2}{2}) + \frac{\rho(1+\delta)}{3\sigma} Re(p_2 - \frac{p_1^2}{2})$$

$$+\frac{\alpha^2\{2[(2\lambda\beta+\lambda-\beta)+1]^2 - 3\mu[2(3\lambda\beta+\lambda-\beta)+1]\}}{12[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]} Req_1^2$$

$$+\frac{\rho^2(1+\delta)[2\eta(\eta+2\gamma_2) - 3\mu\sigma(1+\delta)]}{12\sigma\eta^2} Rep_1^2$$

$$+\frac{\alpha\rho\{2[(2\lambda\beta+\lambda-\beta)+1]^2 - 3\mu(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1]\}}{6\eta[(2\lambda\beta+\lambda-\beta)+1][2(3\lambda\beta+\lambda-\beta)+1]} Rep_1q_1$$

$$\le \frac{2\alpha}{3[2(3\lambda\beta+\lambda-\beta)+1]}(1-R^2) + \frac{2\rho(1+\delta)}{3\sigma}(1-r^2)$$

$$+\frac{\alpha^2\{2[(2\lambda\beta+\lambda-\beta)+1]^2 - 3\mu[2(3\lambda\beta+\lambda-\beta)+1]\}}{3[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]} R^2\cos 2\phi$$

$$+\frac{\rho^2(1+\delta)[2\eta(\eta+2\gamma_2) - 3\mu\sigma(1+\delta)]}{3\sigma\eta^2} r^2\cos 2\theta$$

$$+\frac{2\alpha\rho\{2[(2\lambda\beta+\lambda-\beta)+1]^2 - 3\mu(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1]\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1][2(3\lambda\beta+\lambda-\beta)+1]} rR\cos(\theta+\phi)$$

$$\le \frac{2\alpha}{3[2(3\lambda\beta+\lambda-\beta)+1]}(1-R^2) + \frac{2\rho(1+\delta)}{3\sigma}(1-r^2)$$

$$+\frac{\alpha^2\{3\mu[2(3\lambda\beta+\lambda-\beta)+1] - 2[(2\lambda\beta+\lambda-\beta)+1]^2\}}{3[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]} R^2$$

$$+\frac{\rho^2(1+\delta)[3\mu\sigma(1+\delta) - 2\eta(\eta+2\gamma_2)]}{3\sigma\eta^2} r^2$$

$$+\frac{2\alpha\rho\{3\mu(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1] - 2[(2\lambda\beta+\lambda-\beta)+1]^2\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1][2(3\lambda\beta+\lambda-\beta)+1]} rR = G(r,R).$$

Letting $\alpha, \delta, \nu, \rho$ and μ fixed and differentiating $G(r,R)$ partially when $0 < \alpha \le 1,\ 0 \le \delta < 1,\ 0 \le \nu < 1,\ \rho \ge 1$ and $\mu \ge 1$, we observe that

$$G_{rr}G_{RR} - (G_{rR})^2 = \frac{16\alpha\rho(1+\delta)}{9\sigma[2(3\lambda\beta+\lambda-\beta)+1]}$$

$$+\frac{4\alpha^2\rho^2(1+\delta)\{3\mu[2(3\lambda\beta+\lambda-\beta)+1]-2[(2\lambda\beta+\lambda-\beta)+1]^2\}}{9\sigma\eta^2[(2\lambda\beta+\lambda-\beta)+1]^2}$$

$$\times\frac{\{3\sigma\mu(1+\delta)-2\eta(\eta+2\gamma_2)\}}{[2(3\lambda\beta+\lambda-\beta)+1]}$$

$$-\frac{8\alpha^2\rho(1+\delta)\{3\mu[2(3\lambda\beta+\lambda-\beta)+1]-2[(2\lambda\beta+\lambda-\beta)+1]^2\}}{9\sigma[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}$$

$$-\frac{8\alpha\rho^2(1+\delta)[3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)]}{9\sigma\eta^2[2(3\lambda\beta+\lambda-\beta)+1]}$$

$$-\frac{4\alpha^2\rho^2\{3\mu(1+\delta)[2(3\lambda\beta+\lambda-\beta)+1]-2[(2\lambda\beta+\lambda-\beta)+1]^2\}^2}{9\eta^2[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]^2}<0.$$

Therefore, the maximum of $G(r,R)$ occurs on the boundaries. Thus, the desired inequality follows by observing that

$$G(r,R)\le G(1,1)=\frac{\rho^2(1+\delta)}{3\sigma\eta^2}\{3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)\}$$

$$+\alpha\left\{\frac{3\mu[2(3\lambda\beta+\lambda-\beta)+1]\{\alpha\eta+2\rho[(2\lambda\beta+\lambda-\beta)+1](1+\delta)\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}\right.$$

$$\left.-\frac{2[(2\lambda\beta+\lambda-\beta)+1]^2\{\alpha\eta+2\rho[(2\lambda\beta+\lambda-\beta)+1]\}}{3\eta[(2\lambda\beta+\lambda-\beta)+1]^2[2(3\lambda\beta+\lambda-\beta)+1]}\right\}. \tag{15}$$

The equality is attained when choosing $p_1=q_1=2\mathrm{i}$ and $p_2=q_2=-2$ in (14). □

From the proof of Theorem 1, we can easily have the following corollaries. Letting $\beta=0$ in Theorem 1, we have

Corollary 1. *Let $f(z)$ be given by* (1.1). *If $f(z)\in\Re(\Phi,\Psi;\lambda,0,\alpha,\delta,\nu,\rho)$; $0<\alpha\le 1$, $0\le\lambda\le 1$, $0\le\delta<1$, $0\le\nu<1$, $\rho\ge 1$, $\beta=0$ and $3\mu\sigma(1+\delta)\ge 2\eta(\eta+2\gamma_2)$ where $\eta=\Upsilon_2-\gamma_2(1+\nu)$, $\sigma=\Upsilon_3-\gamma_3(1+\nu)$ and $\mu\ge 0$, then we have*

$$\left|a_3-\mu a_2^2\right|\le\frac{\rho^2(1+\delta)}{3\sigma\eta^2}\{3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)\}$$

$$+\alpha\left\{\frac{3\mu(2\lambda+1)[\alpha\eta+2\rho(\lambda+1)(1+\delta)]-2(\lambda+1)^2[\alpha\eta+2\rho(\lambda+1)]}{3\eta(\lambda+1)^2(2\lambda+1)}\right\}.$$

Letting $\beta=1$ in Theorem 1, we have

Corollary 2. *Let $f(z)$ be given by* (1). *If $f(z)\in\Re(\Phi,\Psi;\lambda,1,\alpha,\delta,\nu,\rho)$; $0<\alpha\le 1$, $0\le\lambda\le 1$, $0\le\delta<1$, $0\le\nu<1$, $\rho\ge 1$, $\beta=1$ and $3\mu\sigma(1+\delta)\ge$*

$2\eta(\eta+2\gamma_2)$ *where* $\eta=\Upsilon_2-\gamma_2(1+\nu)$, $\sigma=\Upsilon_3-\gamma_3(1+\nu)$ *and* $\mu\geq 0$, *then we have*

$$\left|a_3-\mu a_2^2\right|\leq\frac{\rho^2(1+\delta)}{3\sigma\eta^2}\{3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)\}$$

$$+\alpha\left\{\frac{3\mu(8\lambda-1)[\alpha\eta+6\rho\lambda(1+\delta)]-18\lambda^2[\alpha\eta+6\rho\lambda]}{27\eta\lambda^2(8\lambda-1)}\right\}.$$

Letting $\lambda=\beta=1$ *in Theorem* 1, *we have*

Corollary 3. *Let* $f(z)$ *be given by* (1). *If* $f(z)\in\Re(\Phi,\Psi;1,1,\alpha,\delta,\nu,\rho)$; $0<\alpha\leq 1$, $0\leq\delta<1$, $0\leq\nu<1$, $\rho\geq 1$, $\lambda=\beta=1$ *and* $3\mu\sigma(1+\delta)\geq 2\eta(\eta+2\gamma_2)$ *where* $\eta=\Upsilon_2-\gamma_2(1+\nu)$, $\sigma=\Upsilon_3-\gamma_3(1+\nu)$ *and* $\mu\geq 0$, *then we have*

$$\left|a_3-\mu a_2^2\right|\leq\frac{\rho^2(1+\delta)}{3\sigma\eta^2}\{3\mu\sigma(1+\delta)-2\eta(\eta+2\gamma_2)\}$$

$$+\alpha\left\{\frac{21\mu[\alpha\eta+6\rho(1+\delta)]-18[\alpha\eta+6\rho]}{189\eta}\right\}.$$

Remark 1. Letting

$$\Phi(z)=\frac{z}{(1-z)^2},\ \psi(z)=\frac{z}{1-z},\ \lambda=0,\ \beta=0,\delta=0\text{ and }\nu=0$$

in Theorem 1, we have the result given by Darus and Thomas [6].

Remark 2. Letting

$$\Phi(z)=\frac{z}{(1-z)^2},\ \psi(z)=\frac{z}{1-z},\ \lambda=0,\ \beta=0,\ \delta=0,\ \nu=0\text{ and }\alpha=1$$

in Theorem 1, we have the result given by Jahangiri [12].

Remark 3. Letting

$$\lambda=0,\ \beta=0,\ \delta=0\text{ and }\nu=0$$

in Theorem 1, we have the result given by Darus and Hong [8].

Remark 4. Letting

$$\beta=0,\ \delta=0\text{ and }\nu=0$$

in Theorem 1, we have the result given by Darus [7].

Acknowledgements. The author express sincere thanks to the referee for his helpful comments and suggestions.

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Received October 10, 2005

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