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A SUBCLASS OF STARLIKE FUNCTIONS

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Abstract. We introduce a special class of starlike functions, by using the differential subordinations technique due to S.S. Miller and P.T. Mocanu [1], [2]. Some recent results of Gh. Oros and G. Oros [4], [6] are deduced as particular cases.

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1. PRELIMINARIES

Let $U = \{z \in \mathbb{C}, |z| < 1\}$ be the unit disc of the complex plane, and let $H(U)$ be the space of holomorphic functions in U. We consider the following classes of functions:

(1) $H[a,n] = \left\{f \in H(U) : \ f(z) = a + a_n z^n + a_{n+1} z^{n+1} + \dots, \quad z \in U\right\}$;

(2) $\mathcal{A}_n = \left\{f \in H(U) : \ f(z) = z + a_{n+1} z^{n+1} + a_{n+2} z^{n+2} + \dots, \quad z \in U\right\}$, $n \in \mathbb{N}^*$, with $\mathcal{A}_1 = \mathcal{A}$, where $\mathcal{A}$ is the class of *normalized and holomorphic functions* $(f(0) = 0, \ f'(0) = 1)$;

(3) $S^* = \left\{f \in \mathcal{A} : \mathrm{Re}\left[\frac{zf'(z)}{f(z)}\right] > 0, z \in U\right\}$ is the class of *starlike functions* in U;

(4) $S^*(\alpha) = \left\{f \in \mathcal{A} : \mathrm{Re}\left[\frac{zf'(z)}{f(z)}\right] > \alpha, z \in U\right\}$ is the class of *starlike functions of order* α in U, where $\alpha \in [0,1)$;

(5) $M_\alpha = \{f \in \mathcal{A} : \mathrm{Re}\left[J(\alpha, f; z)\right] > 0, z \in U\}$ is the class of α-*convex functions* in U, where $J(\alpha, f; z) = (1-\alpha)\frac{zf'(z)}{f(z)} + \alpha\left(1 + \frac{zf''(z)}{f'(z)}\right)$ and $\alpha \in \mathbb{R}$.

We know that if $\alpha \in \mathbb{R}$ and $f \in M_\alpha$ then f is a starlike function, more precisely we know that if $\alpha \in \mathbb{R}_+$ and $f \in \mathcal{A}$ then $f \in M_\alpha$ if and only if the function $F(z) = f(z)\left[\frac{zf'(z)}{f(z)}\right]^\alpha$ is starlike ([1], [2], [3]). We also know that f is α-*starlike* if the function $F : U \to \mathbb{C}$ with

$$F(z) = f(z)\left(1 - \alpha + \alpha\frac{zf'(z)}{f(z)}\right), \quad z \in U$$

is starlike, where $\alpha \in \mathbb{C}$, [7]. Combining these expressions of the function F, Gh. Oros introduced in [4] and [5] the following subclasses of starlike functions:

$$M_{\alpha,\beta} = \left\{f \in \mathcal{A} : F(z) = f(z)\left(\frac{zf'(z)}{f(z)}\right)^{\alpha(1-\beta)}\left(1 - \alpha + \alpha\frac{zf'(z)}{f(z)}\right)^\beta \in S^*\right\}$$

and

$$M_{\alpha,\frac{1}{2}} = \left\{ f \in \mathcal{A} : F(z) = f(z)\left[\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1-\alpha+\alpha\frac{zf'(z)}{f(z)}\right)\right]^{\frac{1}{2}} \in S^* \right\},$$

where $\alpha, \beta \in \mathbb{R}$ and $\frac{f(z)f'(z)}{z} \neq 0$, $1-\alpha+\alpha\frac{zf'(z)}{f(z)} \neq 0, z \in U$.

He proved that if $\alpha\beta(1-\alpha) \geq 0$ then $M_{\alpha,\beta} \subset S^*$, and clearly, if $\alpha(1-\alpha) \geq 0$ then $M_{\alpha,\frac{1}{2}} \subset S^*$. In this paper we generalize these subclasses of starlike functions, and we prove that these new classes are also subclasses of starlike functions. The next lemma will be used to prove the main result.

Lemma 1.1. *Let Ω be a set in the complex plane $\mathbb{C}$ and let $\psi: \mathbb{C}^3 \times U \mapsto \mathbb{C}$ be a function that satisfy the admissibility condition $\psi(\rho \mathrm{i}, \sigma, \mu + \nu \mathrm{i}; z) \notin \Omega$, where $z \in U$, $\rho, \sigma, \mu, \nu \in \mathbb{R}$ with $\mu + \sigma \leq 0$ and $\sigma \leq -\frac{1}{2}(1+\rho^2)$. If p is analytic in U, with $p(0) = 1$ and*

$$\psi\left(p(z), zp'(z), z^2p''(z); z\right) \in \Omega, \quad z \in U,$$

then $\operatorname{Re} p(z) > 0, \forall z \in U$.

This lemma is a special case of Theorem 9.3.1 of [2], which obtained by taking $q(z) = \frac{1+z}{1-z}$. If we only have $\psi : \mathbb{C}^2 \times U \mapsto \mathbb{C}$, the *admissibility condition* reduces to $\psi(\rho \mathrm{i}, \sigma; z) \notin \Omega, z \in U$ and $\rho, \sigma \in \mathbb{R}$ with $\sigma \leq -\frac{1}{2}(1+\rho^2)$. We remark that if $p \in H[a, n]$ then the last condition in the previous lemma becomes $\sigma \leq -\frac{n}{2}(1+\rho^2)$, and in this paper we have

$$\Omega = \Delta = \{w \in \mathbb{C} : \operatorname{Re} w > 0\}.$$

2. MAIN RESULT

Definition 2.1. Let $f \in \mathcal{A}$, with $\frac{f(z)f'(z)}{z} \neq 0, z \in U$ and let $h \in H(\mathbb{C} \setminus G)$, where G is the set of pols of function h. We say that the function f belongs to the class M_h, if the function $F : U \to \mathbb{C}$ defined by $F(z) = f(z)h\left(\frac{zf'(z)}{f(z)}\right)$ is starlike.

Theorem 2.1. *If h belongs to the class*

$$B_1 = \left\{ g \in H(\mathbb{C} \setminus G) : \operatorname{Re}\left(\frac{g'(z)}{g(z)}\right) > 0, z \in T_1 \right\},$$

then $M_h \subset S^*$, where $T_1 = \{w \in \mathbb{C} : \operatorname{Re} w = 0\}$.

Proof. If we let $p(z) = \frac{zf'(z)}{f(z)}$, $z \in U$, then $p(z) = 1 + a_2 z + \dots$ and from the condition $\frac{f(z)f'(z)}{z} \neq 0$ we deduce that the function p is holomorphic in U. Since F is starlike we have $\operatorname{Re}\left[\frac{zF'(z)}{F(z)}\right] > 0$, $z \in U$, but from the definition we obtain

$$\frac{zF'(z)}{F(z)} = \frac{zf'(z)}{f(z)} + \frac{zh'(w)}{h(w)},$$

where $w = \frac{zf'(z)}{f(z)} = p(z)$. So we obtain the equalities

$$\frac{zF'(z)}{F(z)} = p(z) + zp'(z)\frac{h'(p(z))}{h(p(z))} = \psi\left(p(z), zp'(z)\right).$$

We apply Lemma 1.1 with $\psi(r,s) = r + s\frac{h'(r)}{h(r)}$. It follows that

$$\mathrm{Re}[\psi(\rho\mathrm{i},\sigma)] = \mathrm{Re}\left[\rho\mathrm{i} + \sigma\frac{h'(\rho\mathrm{i})}{h(\rho\mathrm{i})}\right] = \mathrm{Re}\left[\sigma\frac{h'(\rho\mathrm{i})}{h(\rho\mathrm{i})}\right] = \sigma\,\mathrm{Re}\left[\frac{h'(\rho\mathrm{i})}{h(\rho\mathrm{i})}\right] \le 0,$$

because $\sigma \le -\frac{1}{2}(1+\rho^2) < 0$ and $h \in B_1$, i.e. $\mathrm{Re}\left[\frac{h'(\rho\mathrm{i})}{h(\rho\mathrm{i})}\right] \ge 0, \forall\ \rho \in \mathbb{R}$. Using Lemma 1.1, it follows that $\mathrm{Re}\,p(z) > 0$, i.e. f is starlike, which shows that $M_h \subset S^*$. □

Remark 2.1. If we consider the function $h_1 : \Delta_1 \to \mathbb{C}$ defined by $h_1(z) = \prod_{k=1}^{n}(1-\alpha_k+\alpha_k z)^{a_k}$, where $\alpha_k, a_k \in \mathbb{R}$, $k \in \{1,\dots,n\}$ and

$$\Delta_1 = \{z \in \mathbb{C} : 1-\alpha_k+\alpha_k z \ne 0\ \forall k \in \{1,\dots,n\}\},$$

then we obtain the following result:

If $\alpha_k a_k(1-\alpha_k) \ge 0$, for all $k \in \{1,\dots,n\}$ then $M_{h_1} \subset S^*$, where

$$M_{h_1} = \left\{ f \in \mathcal{A} : F(z) = f(z)\prod_{k=1}^{n}\left(1-\alpha_k+\alpha_k\frac{zf'(z)}{f(z)}\right)^{a_k} \in S^* \right\}.$$

To verify this inclusion it is enough to show that $\mathrm{Re}\left(\frac{h_1'(z)}{h_1(z)}\right) \ge 0, \forall\ z \in T_1$. Indeed, we have

$$\mathrm{Re}\left[\frac{h_1'(\rho\mathrm{i})}{h_1(\rho\mathrm{i})}\right] = \mathrm{Re}\left[\sum_{k=1}^{n}\frac{a_k\alpha_k}{1-\alpha_k+\alpha_k(\rho\mathrm{i})}\right] = \sum_{k=1}^{n}\mathrm{Re}\left(\frac{a_k\alpha_k}{1-\alpha_k+\alpha_k\rho\mathrm{i}}\right)$$
$$= \sum_{k=1}^{n}\mathrm{Re}\,\frac{a_k\alpha_k(1-\alpha_k-\alpha_k\rho\mathrm{i})}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2} = \sum_{k=1}^{n}\frac{\alpha_k(1-\alpha_k)a_k}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2}.$$

But $a_k(1-\alpha_k)\alpha_k \ge 0$, for all $k \in \{1,\dots,n\}$. Hence for all $k \in \{1,\dots,n\}$ $\frac{\alpha_k(1-\alpha_k)a_k}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2} \ge 0$, and consequently

$$\sum_{k=1}^{n}\frac{\alpha_k(1-\alpha_k)a_k}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2} \ge 0.$$

Finally, it is obvious that if $\alpha_k \in [0,1]$ and $a_k \in \mathbb{R}_+$ for all $k \in \{1,\dots,n\}$ then $M_{h_1} \subset S^*$.

More generally, if we take the function $h_0 : \Delta_0 \to \mathbb{C}$ defined by $h_0(z) = \prod_{k=1}^{n}(1-\alpha_k+\alpha_k z^p)^{a_k}$, where $\alpha_k, a_k \in \mathbb{R}$ for all $k \in \{1,\dots,n\}$, $p \in \mathbb{N}$ and

$$\Delta_0 = \{z \in \mathbb{C} : 1-\alpha_k+\alpha_k z^p \ne 0, \forall\ k = 1,\dots,n; \alpha_k \in \mathbb{R}, p \in \mathbb{N}\},$$

we obtain the following properties:

(a) if $p \neq 4m+3$ and $\alpha_k a_k(1-\alpha_k) \geq 0$, for all $k \in \{1,\dots,n\}$ then $M_{h_0} \subset S^*$, where $m \in \mathbb{N}$;

(b) if $p \neq 4m+1$ and $\alpha_k a_k(1-\alpha_k) \leq 0$, for all $k \in \{1,\dots,n\}$ then $M_{h_0} \subset S^*$, where $m \in \mathbb{N}$.

In the above property

$$M_{h_0} = \left\{ f \in \mathcal{A} : F(z) = f(z) \prod_{k=1}^{n} \left[1 - \alpha_k + \alpha_k \left(\frac{zf'(z)}{f(z)} \right)^p \right]^{a_k} \in S^* \right\},$$

and we used that

$$\operatorname{Re}\left[\frac{h_0'(\rho \mathrm{i})}{h_0(\rho \mathrm{i})}\right] = \operatorname{Re}\sum_{k=1}^{n} \frac{a_k \alpha_k p (\rho \mathrm{i})^{p-1}}{1-\alpha_k+\alpha_k(\rho \mathrm{i})^p}$$

$$= \begin{cases} 0, & \text{if} \quad p = 4m \\ \displaystyle\sum_{k=1}^{n} \frac{a_k\alpha_k(1-\alpha_k)(4m+1)\rho^{4m}}{(1-\alpha_k)^2+\alpha_k{}^2\rho^{8m+2}}, & \text{if} \quad p = 4m+1 \\ 0, & \text{if} \quad p = 4m+2 \\ \displaystyle -\sum_{k=1}^{n} \frac{a_k\alpha_k(1-\alpha_k)(4m+3)\rho^{4m+2}}{(1-\alpha_k)^2+\alpha_k{}^2\rho^{8m+6}}, & \text{if} \quad p = 4m+3 \end{cases}$$

Now we define the following class of functions:

Definition 2.2. Let $\alpha_k \in [0,1]$, $a_k \in \mathbb{R}_+$, $\forall\ k \in \{1,\dots,n\}$ and let $f \in \mathcal{A}$, with $\frac{f(z)f'(z)}{z} \neq 0, z \in U$. Assume that for all $k \in \{1,\dots,n\}$, $1-\alpha_k+\alpha_k\frac{zf'(z)}{f(z)} \neq 0$, $z \in U$. We say that the function f belongs to the class $\widetilde{M}_{\alpha_k,a_k}$ if the function

$$F : U \to \mathbb{C}, \qquad F(z) = f(z) \prod_{k=1}^{n} \left(1 - \alpha_k + \alpha_k \frac{zf'(z)}{f(z)} \right)^{a_k}$$

is starlike of order $\sum_{k=1}^{n} C(\alpha_k, a_k)$, where

$$C(\alpha_k, a_k) = \begin{cases} \frac{\alpha_k}{\alpha_k - 1} \cdot \frac{a_k}{2}, & \text{if} \quad \alpha_k \in [0, \frac{1}{2}] \\ \frac{\alpha_k - 1}{\alpha_k} \cdot \frac{a_k}{2}, & \text{if} \quad \alpha_k \in [\frac{1}{2}, 1]. \end{cases}$$

Theorem 2.2. *For any $\alpha_k \in [0,1]$, $a_k \in \mathbb{R}$ and $k \in \{1,\dots,n\}$, the inclusion $\widetilde{M}_{\alpha_k,a_k} \subset S^*$ holds.*

Proof. We proceed as in the Theorem 2.1. Let $p(z) = \frac{zf'(z)}{f(z)}$. From the definition we see that p is analytic in U, and since F is starlike of order $\sum_{k=1}^{n} C(\alpha_k, a_k)$ then we have the following relation:

$$\operatorname{Re}\left[\frac{zF'(z)}{F(z)} - \sum_{k=1}^{n} C(\alpha_k, a_k)\right] > 0, \quad \forall\ z \in U.$$

It follows that

$$\operatorname{Re}\left[\psi(p(z), zp'(z))\right] = \operatorname{Re}\left[\frac{zF'(z)}{F(z)} - \sum_{k=1}^{n} C(\alpha_k, a_k)\right]$$

$$= \operatorname{Re}\left[p(z) + zp'(z)\sum_{k=1}^{n}\frac{a_k\alpha_k}{(1-\alpha_k)+\alpha_k p(z)} - \sum_{k=1}^{n} C(\alpha_k, a_k)\right] > 0.$$

We have

$$\begin{aligned}
\operatorname{Re}\psi(\rho\mathrm{i}, \sigma) &= \operatorname{Re}\left[\rho\mathrm{i} + \sigma\sum_{k=1}^{n}\frac{a_k\alpha_k}{(1-\alpha_k)+\alpha_k(\rho\mathrm{i})} - \sum_{k=1}^{n} C(\alpha_k, a_k)\right] \\
&= \sum_{k=1}^{n}\frac{a_k\alpha_k(1-\alpha_k)\sigma}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2} - \sum_{k=1}^{n} C(\alpha_k, a_k) \\
&\le -\sum_{k=1}^{n}\frac{a_k\alpha_k(1-\alpha_k)(1+\rho^2)}{2\left[(1-\alpha_k)^2+{\alpha_k}^2\rho^2\right]} - \sum_{k=1}^{n} C(\alpha_k, a_k) \\
&= -\sum_{k=1}^{n}\left\{\frac{a_k\alpha_k(1-\alpha_k)(1+\rho^2)}{2\left[(1-\alpha_k)^2+{\alpha_k}^2\rho^2\right]} + C(\alpha_k, a_k)\right\} \le 0,
\end{aligned}$$

because $\alpha_k \in [0,1]$, $a_k \in \mathbb{R}_+$, for all $k \in \{1,\dots,n\}$ and

$$\frac{a_k\alpha_k(1-\alpha_k)(1+\rho^2)}{2\left[(1-\alpha_k)^2+{\alpha_k}^2\rho^2\right]} + C(\alpha_k, a_k)$$

$$= \begin{cases} \dfrac{a_k}{2}\cdot\dfrac{2\alpha_k-1}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2}\cdot\dfrac{\alpha_k}{\alpha_k-1}\cdot\rho^2, & \text{if} \quad \alpha_k \in [0,\frac{1}{2}] \\[2ex] \dfrac{a_k}{2}\cdot\dfrac{2\alpha_k-1}{(1-\alpha_k)^2+{\alpha_k}^2\rho^2}\cdot\dfrac{1-\alpha_k}{\alpha_k}, & \text{if} \quad \alpha_k \in [\frac{1}{2},1] \end{cases}$$

Hence by Lemma 1.1 we obtain that $\operatorname{Re} p(z) > 0$, i.e. $f \in S^*$. □

Remark 2.2. (1) If we take $k = 2, \alpha_1 = 1, \alpha_2 = \alpha, a_1 = \alpha(1-\beta)$ and $a_2 = \beta$ in Theorem 2.2 and in the last Remark we obtain the results of Gh. Oros ([4], [5]). On the other hand, take the function

$$h_2 : \Delta_2 \to \mathbb{C},\ \Delta_2 = \{z \in \mathbb{C}^* : 1-\alpha+\alpha z \ne 0\}$$

defined by $h_2(z) = z^{\alpha(1-\beta)}(1-\alpha+\alpha z)^\beta$. In this case we have that $M_{h_2} = M_{\alpha,\beta}$, the class of starlike functions defined by Gh. Oros.

(2) If we take the function

$$h_3 : \Delta_3 \to \mathbb{C},\ \Delta_3 = \{z \in \mathbb{C} : 1-\alpha+\alpha z \ne 0\}$$

defined by $h_3(z) = 1-\alpha+\alpha z$, $\alpha \in \mathbb{C}$ then the function $F_3 : U \to \mathbb{C}$ is given by $F_3(z) = f(z)\left[1-\alpha+\alpha\frac{zf'(z)}{f(z)}\right]$. We obtain in this case the class $M_{h_3} = M_{\alpha,1}$, of α-starlike functions defined by N. N. Pascu ([7]).

(3) If we take $h_4 : \mathbb{C}^* \to \mathbb{C},\ h_4(z) = z^\alpha$, then we obtain the class of α-convex functions and $F_4 : U \to \mathbb{C}$ is given by $F_4(z) = \left[\frac{zf'(z)}{f(z)}\right]^\alpha$, $M_{h_4} = M_{\alpha,0} = M_\alpha$, defined by P. T. Mocanu ([1], [2], [3]).

(4) If we take $h_5 : \mathbb{C}^* \to \mathbb{C},\ h_5(z) = z$, then $F_5 : U \to \mathbb{C}$ is given by $F_5(z) = zf'(z)$ and by the *Alexander duality theorem* $M_{h_5} = M_{1,\beta} = K$, the class of convex functions.

(5) If we take $h_6 : \mathbb{C} \to \mathbb{C},\ h_6(z) = 1$, then $F_6 : U \to \mathbb{C}$ is given by $F_6(z) = f(z)$, and $M_{h_6} = M_{0,\beta} = S^*$ is the class of starlike functions.

(6) If $h_7 : \mathbb{C} \to \mathbb{C}$ is defined by $h_7(z) = k\exp\left(\frac{z^2}{2}+cz\right)$, where $k \in \mathbb{C}$, $c \geq 0$, then $M_{h_7} \subset S^*$, or more generally $h_8, h_9, h_{10} : \mathbb{C} \to \mathbb{C}$, defined by $h_8(z) = k\exp\left(\sum_{p=1}^{n}\frac{z^{2p}}{2p}+cz\right)$, $h_9(z) = k\exp\left(\sum_{p=1}^{\infty}\frac{z^{2p}}{2p}+cz\right)$, $h_{10}(z) = k\exp\left(\cos z+cz\right)$. It is easy to show that $h_7, h_8, h_9, h_{10} \in B_1$. For example, for h_9:

$$\operatorname{Re}\left[\frac{h_9'(\rho \mathrm{i})}{h_9(\rho \mathrm{i})}\right] = \operatorname{Re}\left[\sum_{p=1}^{\infty}(\rho \mathrm{i})^{2p-1}+c\right] = c \geq 0.$$

So in these remarks we see that $M_{h_i} \subset S^*$, for all $i \in \{2,\ldots,10\}$. We close this paper with the observation that if we consider a function in $\mathcal{A}_n$, then we have analogous properties.

Definition 2.3. Let $f \in \mathcal{A}_n$, with $\frac{f(z)f'(z)}{z} \neq 0, z \in U$ and let $h \in H(\mathbb{C}\setminus G)$, where G is the set of pols of function h. We say that the function f belongs to the class $M_h^n(\alpha)$ if the function $F : U \to \mathbb{C}$ defined by the following relation $F(z) = f(z)h\left(\frac{zf'(z)}{f(z)}\right)$ is starlike of order α, $\alpha \in [0,1)$.

Theorem 2.3. *If the function h belongs to the following class of functions*

$$B_2 = \left\{g \in H(\mathbb{C}\setminus G) : \operatorname{Re}\left(\frac{g'(z)}{g(z)}\right) > 0, z \in T_2\right\},$$

then $M_h^n(\alpha) \subset S^*$, $\forall\ \alpha \in [0,1)$, *where* $T_2 = \{w \in \mathbb{C} : \operatorname{Re} w \in (0,1]\}$.

Proof. Let $\frac{zf'(z)}{f(z)} = (1-\alpha)p(z)+\alpha$, $z \in U$. From the condition we deduce that p is holomorphic in U and $p \in H[1,n]$. Let $\psi\left(p(z), zp'(z)\right) = \left[\frac{zF'(z)}{F(z)}\right]-\alpha$, where

$$\left[\frac{zF'(z)}{F(z)}\right]-\alpha = (1-\alpha)p(z)+(1-\alpha)zp'(z)\frac{h'(\alpha+(1-\alpha)p(z))}{h(\alpha+(1-\alpha)p(z))}.$$

Since F is starlike of order α, it follows that $\operatorname{Re}\psi\,(p(z), zp'(z)) > 0, z \in U$. We have

$$\operatorname{Re}[\psi(\rho\mathrm{i},\sigma)] = \operatorname{Re}\left[(1-\alpha)\rho\mathrm{i} + (1-\alpha)\sigma\frac{h'(1-\alpha+\alpha\rho\mathrm{i})}{h(1-\alpha+\alpha\rho\mathrm{i})}\right]$$
$$= (1-\alpha)\sigma\operatorname{Re}\left[\frac{h'(1-\alpha+\alpha\rho\mathrm{i})}{h(1-\alpha+\alpha\rho\mathrm{i})}\right] \le 0,$$

because $h \in B_2$ and $\sigma \le -\frac{n}{2}(1+\rho^2) < 0$. By Lemma 1.1 we have $\operatorname{Re} p(z) > 0$. Then

$$\operatorname{Re}\left[\frac{zf'(z)}{f(z)}\right] = \operatorname{Re}[(1-\alpha)p(z)+\alpha] > \alpha,$$

which shows that $M_h^n(\alpha) \subset S^*$, $\forall\ \alpha \in [0,1)$. □

REMARK 2.3. For $\alpha = 0, n = 1$ we deduce Theorem 2.1, and if we take the function $h(z) = \sqrt{\frac{z(z+1)}{2}}$, $h : \mathbb{C} \setminus \{-1,0\} \to \mathbb{C}$, we obtain the result of G. Oros [6].

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