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BOOK REVIEWS

John E. Jayne and C. Ambrose Rogers, *Selectors,* Princeton University Press, Princeton and Oxford 2002, XIV + 167 pp., ISBN: 0-691-09628-7.

A selector for a set-valued map $F : X \to Y$ such that $F(x) \neq \emptyset$, $\forall x \in X$, is a map $f : X \to Y$ such that $f(x) \in F(x)$, $\forall x \in X$. Here X, Y are arbitrary sets and, in this general form, the existence of selectors is equivalent to the axiom of choice. Supposing that X, Y have some additional structures, e.g., they are topological spaces or measure spaces, one looks for selectors having some additional properties too, of topological nature (continuity, being of first Baire or of first Borel class) or being measurable. A function f between two topological spaces X, Y is called of first Baire class if it is the pointwise limit of a sequence of continuous functions, and of first Borel class if $f^{-1}(U)$ is an F_σ-subset of X for any open subset U of Y. Of course, in order to prove the existence of such selectors, the set-valued map F and the spaces X and Y must satisfy some supplementary requirements. For instance, the classical Michael's selection theorem asserts that any lower semi-continuous set-valued map from a paracompact space X to a Banach space Y, which takes only nonempty closed convex values, has a continuous selector. A proof of this theorem, along with that of Kuratowski and Ryll-Nardzewski theorem on the existence of selectors of first Borel class for a lower semi-continuous set-valued map from a metric space to a complete separable metric space, are given in the first chapter of the book *Classical results.*

The second chapter, *Functions that are constant on the sets of a disjoint discretely σ-decomposable family of F_σ-sets*, contains some rather technical results, needed in Chapters 3–7 where the selectors are obtained as limits of functions of this kind.

The main selection theorems are proved in Chapters 3, *Selectors for upper semi-continuous functions with nonempty compact values,* 4, *Selectors for compact sets,* and 6, *Selectors for upper semi-continuous set-valued maps with nonempty values that are otherwise arbitrary.* Chapter 6 is based on some striking results obtained by S. S. Srivatsa in 1985–1986, during a visit at the University College, London, and published much later (Trans. Amer. Math. Soc., 337 (1993), 609–624). A key role in the proofs of selection theorems is played by the notions of fragmentability and σ-fragmentability in Banach spaces, notions due to the authors of the book (Acta Math., 155 (1985), 41–79).

In Chapter 5, *Applications,* the selection theorems proved in the previous chapters are applied to some set-valued maps which have geometric origins. These are: the maximal monotone operators and the subdifferential operators of continuous convex functions, two sort of attainment maps (maps taking as values support points, respectively support functionals), and the metric projection maps. Although these maps were intensively studied by many reputed mathematicians (R. T. Rockafellar, P. Kenderov, R. R. Phelps, a.o.), the authors of the present monograph succeeded to prove the existence of selectors of first Baire class (see loc. cit.). It is worth mentioning that, based on these results, M. Raja (Math. Nachr., 254/255 (2003), 27–34) proved recently a very general result on the proximinality of the space $L_1(S,Y)$ in $L_1(S,X)$.

The last chapter of the book, Chapter 7, *Further applications,* contains some selection theorems centered around the notion of Asplund space. In this chapter some locally uniformly convex renorming results are taken for grant, without proofs. Excepting this chapter, the rest of the book is fairly self-contained, with complete proofs.

The result is a fine book, written in a pleasant and accurate style, that incorporates a lot of deep and useful results on selectors and their applications, including many belonging to the authors. It appeals to a large audience – people interested in analysis, functional analysis, in optimization and control, or working in other areas where set-valued maps and selectors are used.

S. Cobzaş

Elias M. Stein and Rami Shakarchi, *Fourier Analysis – An Introduction,* Princeton Lectures in Analysis, Vol. I, Princeton University Press, Princeton and Oxford 2003, XVI + 311 pp., ISBN: 0-691-11384-X.

This is the first part of a four-volume treatise on analysis centered around Fourier analysis, a choice motivated by the role it played in the development of the subject and by its occurrence in many areas of the modern analysis. The books are based on a series of four one-semester courses taught at Princeton University beginning in the spring of 2000, and having as main objective to emphasize the interplay between various parts of analysis and, at the same time, to illustrate the wide applicability of ideas of analysis to other fields of mathematics and science.

The first volume is devoted to an elementary exposition of basic principles of Fourier analysis. The prerequisites are kept at minimum and comprise only familiarity with Riemann integration, limits, series, differentiability, and some linear algebra. The basic results on Riemann integration on $\mathbb{R}$ (with proofs) and on $\mathbb{R}^d$ (without proofs) are presented in an appendix. The aim of this introductory volume is to make the beginning student acquainted with the basic ideas of the subject, before passing to more sophisticated and technical topics, and to motivate the study by some non-trivial applications. Even at this level the authors succeed to present some deep and very interesting applications, such as the Hurwitz solution to the isoperimetric problem in the plane, Weil's equidistribution theorem, an example of continuous nowhere differentiable function, Dirichlet's theorem on the infinity of prime numbers in arithmetic progression, a discussion on Heisenberg uncertainty principle.

The first chapter of the book, *The genesis of Fourier analysis,* presents at a heuristic level the evolution of Fourier analysis and some of the problems lying at its origins – the vibrating string and the propagation of heat equations. The other chapters are devoted to the exposition of basic results in Fourier analysis. These are: 2. *Basic principles of Fourier series,* 3. *Convergence of Fourier series,* 4. *Some applications of Fourier series,* 5. *The Fourier transform on* $\mathbb{R}$, 6. *The Fourier transform on* $\mathbb{R}^d$ (including a discussion on the Radon transform in $\mathbb{R}^2$ and $\mathbb{R}^3$), 7. *Finite Fourier transform* (dealing with fast Fourier transform and Fourier analysis on finite abelian groups), 8. *Dirihlet's theorem.*

One of the authors (E.M.S.) is a well known specialist in harmonic analysis and his previous books became standard references in the area. This introductory book shows that the study of Fourier analysis deserves the effort required to handle the various areas of mathematics involved. By the completion with the other three volumes, II. *Complex analysis,* III. *Measure theory, Lebesgue integration, and Hilbert spaces,* and IV. *A selection of further topics* (according to the foreword of this book), this treatise will do a great service to the mathematical community.

S. Cobzaş

Vladimir Müller, *Spectral Theory of Linear Operators (and spectral Systems in Banach Algebras),* Operator Theory: Advances and Applications, Vol. **139**, Birkhäuser Verlag, Basel-Boston-Berlin 2003, X+381 pp., ISBN: 3-7643-6912-4.

The book is devoted to the basic results in spectral theory in Banach algebras for both single elements and for n-tuples of commuting elements, with emphasis on the spectral theory of operators on Banach and Hilbert spaces. The unifying idea, allowing to present in an axiomatic and elementary way various types of spectra – the approximate point spectrum, Taylor spectrum, local spectrum, essential spectrum, etc. – is that of regularity in a Banach algebra. A regularity is a subset R of a unital Banach algebra $\mathcal{A}$ having some nice properties as : $ab \in R \iff a, b \in R$; $e \in R$; $\mathrm{Inv}(\mathcal{A}) \subset R$. The spectrum of an element $a \in \mathcal{A}$ with respect to a regularity R is defined by $\sigma_R(a) = \{\lambda \in \mathbb{C} : a - \lambda e \notin R\}$. For $R = \mathrm{Inv}(\mathcal{A})$ one obtains the usual spectrum $\sigma(a)$ of the element a. This notion, introduced and studied by the author and V. Kordula (Studia Math. **113** (1995), 127–139), is sufficiently general to cover many interesting cases of spectra but, at the same time, sufficiently strong to have non-trivial consequences as, e.g., the spectral mapping theorem.

The first chapter of the book, Ch. I, *Banach algebras*, presents the basic results on spectral theory in Banach algebras, including the axiomatic theory of spectrum via regularities. A special attention is paid to approximate point spectrum and its connection with removable and non-removable ideals. In the second chapter, Ch. II, *Operators,* these notions and results are specified to the very important case of operators on Banach and Hilbert spaces. Chapter III, *Essential spectrum,* is concerned with spectra in the Calkin algebra $\mathcal{B}(X)/\mathcal{K}(X)$, and with Fredholm and Browder operators. Although having a rather involved definition, the Taylor spectrum for commuting finite systems of operators seems to be the natural extension of ordinary spectrum for single operators, due mainly to the existence of functional calculus for functions analytic in a neighborhood of it. The presentation of Taylor functional calculus is done in the fourth chapter in an elementary way, without the use of sheaf theory and cohomological methods, following the ideas from a paper by the author of the book (Studia Math. **150** (2002), 79–97).

The last chapter of the book, Ch. IV, *Orbits and capacity*, is concerned with the study of orbits, meaning sequences $\{T^n x : n = 0, 1, ...\}$ in Banach or Hilbert spaces, a notion closely related to those of local spectral radius and capacity of an operator. Some Baire category results of the author on the boundedness of the orbit are included.

The book is clearly written and contains a lot of material, some of it appearing for the first time in book form. At the same time, the author tried successfully to keep the presentation at an elementary level, the prerequisites being basic functional analysis, topology and complex function theory (some needed results are collected in an Appendix at the end of the book).

The book, or parts of it, can be used for graduate or postgraduate courses, or as a reference text.

S. Cobzaş

Sergiu Kleinerman and Francesco Nicolò, *The Evolution Problem in General Relativity,* Progress in Mathematical Physics, Vol. 25, Birkhäuser Verlag, Boston-Basel-Berlin 2003, XXII+385 pp., ISBN: 3-7643-4254-4 and 0-8176-4254-4.

From the Preface: "The aim of the present book is to give a new self-contained proof of the global stability of the Minkowski space, given in D. Christodoulou and S. Kleinerman, *The global nonlinear stability of the Minkowski space,* Princeton Mathematical Series, Vol. **41**, Princeton 1993 (Ch-Kl). We provide a new self-contained proof of the main part of that result, which concerns the full solution of the radiation problem in vacuum, for arbitrary asymptotically flat initial data sets. This can be also interpreted as a proof of the global stability of the external region of Scwarzschild spacetime.

The proof, which is a significant modification of the argument in Ch-Kl, is based on a *double null foliation* of spacetime instead of the *mixed null-maximal foliation* used in Ch-Kl. This approach is more naturally adapted to the radiation features of the Einstein equations and leads to important technical simplifications."

The book is fairly self-contained, the basic notions from differential geometry being reviewed in the first chapter. This chapter contains also a review of known results on Einstein equations and initial data value problems in general relativity, and the formulation of the main result.

The rest of the book is devoted to technical preparations for the proof, and to the proof of the main result. These chapters are headed as follows: 2. *Analytic methods in the study of the initial value problems,* 3. *Definitions and results,* 4. *Estimates for the connection coefficients,* 5. *Estimates for the Riemann curvature tensor,* 6. *The error estimates,* 7. *The initial hypersurface and the last slice,* and 8. *Conclusions.* This last chapter contains a rigorous derivation of the Bondi mass as well as of the connection between the Bondi mass and the ADM mass.

This important monograph, presenting the detailed proof of an important result in general relativity, is of great interest to researchers and graduate students in mathematics, mathematical physics, and physics in the area of general relativity.

Paul A. Blaga

Jürgen Moser, *Selected Chapters in the Calculus of Variations*, (Lectures notes by Oliver Knill), Lectures in Mathematics, ETH Zürich, Birkhäuser Verlag, Basel-Boston-Berlin 2003, XVI+132 pp., ISBN: 3-7643-2185-7.

This book is based on the lectures presented by J. Moser in the spring of 1998 at the Eidgenössische Technische Hochschule (ETH) Zürich. The course was attended by students in the 6th and 8th semesters, by some graduate students and visitors from ETH. The German version of the notes was typed in the summer of of 1998 and J. Moser carefully corrected it the same year in September. A translation was done in 2002 and figures were included, but the original text remained essentially unchanged.

The lectures are concerned with a new development in the calculus of variations – the so called Aubry-Mather Theory. It has its origins in the research of the theoretical physicist S. Aubry on the motion of electrons in two dimensional crystal, and in that of J. Mather on monotone twist maps, appearing as Poincaré maps in mechanics. They were studied by G. Birkhoff in 1920s, but it was J. Mather in 1982 who succeeded to make substantial progress proving the existence of a class of closed invariant subsets, called now Mather sets. The unifying topic of both Aubry and Mather approaches is that of some variational principles, a point that the book makes very clear.

The material is grouped in three chapters: 1. *One-dimensional variational problems;* 2. *Extremal fields and global minimals;* 3. *Discrete systems, Applications.*

The first chapter collects the basic results from the classical theory, the notion of extremal fields being a central one. In the second chapter the variational problems on the 2-dimensional torus are investigated, leading to the notion of Mather set. In the last chapter the connection with monotone twist maps is made, as a starting point of Mather's theory, and the discrete variational problems lying at the basis of Aubry's theory are presented.

The aim of the book is not to present the things in their greatest generality, but rather to emphasize the relations of the newer developments with classical notions.

The progress made in the area since 1998 is shortly presented in an Appendix along with some additional literature.

The book is ideal for advanced courses in the calculus of variations and its applications.

J. Kolumbán

Steven G. Krantz and Harold R. Parks, *A Primer of Real Analytic Functions*, Birkhäuser Advanced Texts, Birkhäuser Verlag, Boston-Basel-Berlin 2002, XII+205 pp., ISBN: 0-8175-4264-1.

Complex analytic functions of one or several complex variables are presented in a lot of books, at introductory level and at advanced as well.

Their older and poorer relatives – the real analytic functions – having totally different features, found their first book treatment in the first edition of the present book, published by Birkhäuser in 1992. Real analytic functions are an essential tool in the study of embedding problem for real analytic manifolds. They have also applications in PDEs and in other areas of analysis.

With respect to the first edition, beside the revision of the presentation, some new material on topologies on spaces of real analytic functions and on the Weierstrass preparation theorem, has been added.

The basic results on real analytic functions are presented in the first two chapters: Ch. 1 *Elementary properties,* and Ch. 2 *Multivariable calculus of real analytic functions*, including implicit and inverse function theorems, Cauchy-Kowalewski theorem.

Chapters 3, *Classical topics* and 4, *Some questions in hard analysis*, contain more advanced topics as Besicovitch's theorem, Whitney's extension and approximation theorems, quasi-analytic classes and Gevrey classes, Puiseux series.

Ch. 5, *Results motivated by PDEs*, is concerned with topics as division of distributions, the FBI transform (FBI comes here from the name of mathematical physicists Fourier, Bros and Iagnolitzer), and Paley-Wiener theorem.

The last chapter, Ch. 6, *Topics in geometry*, contains a discussion of some deep and difficult results as embedding of real analytic manifolds, sub- and semi-analytic sets, the structure theorem for real analytic varieties.

Bringing together results scattered in various journals or books and presenting them in a clear and systematic manner, the book is of interest first of all for analysts, but also for applied mathematicians and for researcher in real algebraic geometry.

P. T. Mocanu

Erik M. Alfsen and Frederic W. Shultz, *Geometry of State Spaces of Operator Algebras,* Mathematics: Theory and Applications, Birkhäuser Verlag, Boston-Basel-Berlin 2003, XIII+467 pp., ISBN: 0-8176-4319-2 and 3-7643-4319-2.

The aim of the present book is to give a complete geometric description of the state spaces of operator algebras, meaning to give axiomatic characterizations of those convex sets that are state spaces of C^*-algebras, von Neuman algebras, and of their nonassociative analogs – JB-algebras and JBW-algebras. A previous book by the same authors – *State spaces of operator algebras – basic theory, orientations and C^*-products,* published by Birkhäuser in 2001, contains the necessary prerequisites on C^*-algebras and von Neumann algebras but, for the convenience of the reader, these results are summarized in an appendix at the end of the present book with exact references to previous one for proofs.

The problem of the characterization of state spaces of operator algebras was raised in the early 1950s and was completely solved by the authors of the present book in Acta Mathematica, **140** (1978), 155–190, and **144** (1980), 267–305 (the second paper has also H. Hanche-Olsen as co-author). Although the axioms for state spaces are essentially geometric, many of them have physical interpretations. The authors have included a series of remarks concerning these interpretations along with some historical notes.

The book is divided into three parts. Part I (containing Chapters 1 through 6) can serve as an introduction for novices to Jordan algebras and their states. Jordan algebras were originally introduced as mathematical model for quantum mechanics (in 1934 by P. Jordan, J. von Neumann and E. Wigner), starting from the remark that the set of observables is closed under Jordan multiplication, but not necessarily under associative multiplication. Part II (Chapters 7 and 8) develops the spectral theory for affine functions on convex sets. The functional calculus developed in this part reflects a key property of the subalgebra generated by a single element and, physically, it represents the application of a function to the outcome of an experiment. Part III (Chapters 9, 10, 11) gives the axiomatic characterization of operator algebra state spaces and explain how the algebras can be reconstructed from their state spaces.

This valuable book, together with the previous one on C^*-algebras, presents in a manner accessible to a large audience, the complete solution to a long standing problem, available previously only in research papers, whose understanding requires a solid background from the readers.

It is aimed to specialists in operator algebras, graduate students and mathematicians working in other areas (mathematical physics, foundation of quantum mechanics).

S. Cobzaş

Emmanuele DiBenedetto, *Real Analysis*, Birkhäuser Advanced Texts, Birkhäuser Verlag, Boston-Basel-Berlin, 2002, XXIV+485 pp., ISBN: 0-8175-4231-5.

The aim of this book is to present at graduate level the basic results in real analysis, needed for researchers in applied analysis – PDEs, calculus of variations, probability, and approximation theory. Assuming only the knowledge of the basic results about the topology of $\mathbb{R}^N$, series, advanced differential calculus and algebra of sets, the author develops the whole machinery of real analysis bringing the reader to the frontier of current research.

The emphasis is on measure and integration in $\mathbb{R}^N$, meaning Lebesgue and Lebesgue-Stieltjes measures, Radon measures, Hausdorff measure and dimension. The topological background, including Tihonov compactness theorem, Tietze and Urysohn theorems, is developed with full proofs. The specific of the book is done by the treatment of some more specialized topics than those usually included in introductory courses of real analysis. Between these topics I do mention a detailed presentation of covering theorems of Vitali and Besicovitch, the Marcinckiewicz integral, the Legendre transform, the Rademacher theorem on the a.e. differentiability of Lipschitz functions. Fine topics, as a.e. differentiability of functions with bounded variation and of absolutely continuous functions and the relation with the integral, are worked out.

The spaces L^p are also presented in details in Chapter V – completeness, uniform convexity (via Hanner's inequalities), duality, weak convergence, compactness criteria. The next chapter of the book (Ch. VI) contains a brief introduction to abstract Banach and Hilbert spaces. Distributions, weak differentials and Sobolev spaces are presented in Chapter VII.

The last two chapters of the book, Chapters VIII and IX, contains more specialized topics as maximal functions and Fefferman-Stein theorem, the Calderón-Zygmund decomposition theorem, functions of bounded mean oscillation (BMO), Marcinkiewicz interpolation theorem, embedding theorems for Sobolev spaces, Poincaré inequality, Morrey spaces.

Each chapter is completed by a set of exercises and problems that add new features and shed new light on the results from the main text.

Bringing together, in a relatively small number of pages, important and difficult results in real analysis that are of current use in application to PDEs, Fourier and harmonic analysis and approximation, this valuable book is of great interest to researchers working in these areas, but it can be used for advanced graduate courses in real analysis as well.

S. Cobzaş

Bernhelm Booss-Bavnek and Jens Høyrup Editors, *Mathematics and War*, Birkhäuser Verlag, Basel-Boston-Berlin, 2003, VIII+416 pp., ISBN: 3-7643-1634-9.

The volume contains some of the contributions delivered at the International Meeting on Mathematics and War, held in Karlskrona, Sweden, from 29 to 31 august, 2002, together with some invited papers. The idea was to bring together mathematicians, historians, philosophers and military, to discuss some of the interconnections between warfare and mathematics. As it is well known after the World War II (WW II), there has been a strong mathematization of warfare and of the concepts of modern war, which in its turn deeply influenced the development of some areas of mathematics.

The papers included in the volume deal with topics ranging from historical, philosophical, ethical aspects of the problem, to more technical aspects like the functioning of weapons, the actual planning of war and information warfare. The perspectives the authors approache the treated theme also differ from one to other – some papers are written from a pacifist point of view (more or less explicitly), while others are not. Some of the papers are dealing with history, but focused on the last sixty years, e.g., WW II and the Kosovo war.

The volume is organized in four parts: I. *Perspectives from mathematics,* II. *Perspectives from the military,* III. *Ethical issues,* IV. *Enlightenment perspectives.*

The first part contains studies on military work in mathematics 1914-1945 (R. Siegmund-Schultz), on the Enigma code breaking work in Poland (E. Rakus-Anderson), on the defence work of A. N. Kolmogorov (A. N. Shiryaev), on the discovery of maximum pricilple by Lev Pontryagin (R. V. Gamkrelidze), and on the mathematics and war in Japan (S. Fukutomi).

The second part is written by military and deals with topics as information warfare (U. Bernhard and I. Ruhmann), the exposure of civilians under the modern "safe" warfare (E. Schmägling), duels of systems and forces (H. Löfstedt).

The third part is concerned with N. Bohr's and A. Turing's involving in military research (I. Aaserud and A. Hodges, respectively), and K. Ogura and the "Great Asia War" (T. Makino).

The last part contains two studies – one on mathematical thinking and international law (I. M. Jarvard), and one on modeling the conflict and cooperation (J. Scheffran).

The aim of the volume is to draw the attention of scientists, military and philosophers on the dramatic consequences that the use of science, particularly of mathematics, for military purposes can have on the development of humanity, and to trace some possible way of preventing this disaster.

P. T. Mocanu

Harvey E. Rose, *Linear Algebra. A Pure Mathematical Approach.* Birkhäuser Verlag, Basel-Boston-Berlin, 2002, XIV+250 pp., Softcover, ISBN 3-7643-6792-X.

The book is a comprehensive approach of the topic linear algebra. It offers a modern point of view over the matter, some applications being also included there. Although the book is self-contained, the (ideal) reader is supposed to have elementary knowledge about matrix and determinants.

The book covers the usual topic in linear algebra, namely vector spaces, bases and dimension, direct sums, linear maps and their representation by matrices, the theorems of Hamilton–Cayley and Jordan, and a briefly exposition of the needed prerequisites, as groups and fields. In addition it is included a number of applications to group representation theory, mathematical coding theory, geometry of quadratic forms, quaternions and Cayley numbers. Even if some of these applications led us to applied mathematics (e.g. coding theory), the approach is, as says the title, puremathematical.

A special attention should be accorded to the problems which are provided after each chapter. These problems are carefully selected, some of them being only calculatory exercises, for which the author recommends the usage of a computer algebra package, another emphasizing structural aspects of the subject. The answers, more or less detailed, to all the problems are provided as well.

Through the above mentioned openings "both inside and outside mathematics", the book fills a lack in the literature about a very old and venerable subject. It will be useful to the students, who need linear algebra in their preparation, e.g. in mathematics, computer science or natural sciences. It may be also used by teachers in their activity. Moreover it offers a good introduction in abstract mathematics for all those interested.

Ciprian Modoi