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SOME PROPERTIES OF WEAKLY QUASICONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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Abstract. In this paper, we obtain several new characterizations of weakly quasi continuous functions in bitopological spaces by using preopen sets and also we study some properties of such functions.

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Key words. Bitopological spaces, weakly quasicontinuous, preopen sets.

1. INTRODUCTION

The notion of weakly semi-continuous functions in bitopological spaces was initiated by Khedr [7]. Popa and Noiri [16] introduced the notion of weakly quasicontinuous functions in bitopological spaces. In [16], they obtained several characterizations of weakly quasicontinuous functions in bitopological spaces and showed that weak quasicontinuity is equivalent to weak semi-continuity in bitopological spaces. Recently, Nasef [13] has introduced the notion of almost quasicontinuous functions in bitopological spaces.

The purpose of the present paper is to obtain new characterizations of weak quasicontinuity in bitopological spaces by using preopen sets and also to obtain some sufficient conditions for weakly quasicontinuous functions to be quasicontinuous or almost quasicontinuous in bitopological spaces.

2. PRELIMINARIES

Throughout the present paper, (X, τ_1, τ_2) (resp. (X, τ)) denote a bitopological (resp. topological) space. Let (X, τ) be a topological space and A be a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. The closure of A and the interior of A with respect to τ_i are denoted by $i\mathrm{Cl}(A)$ and $i\mathrm{Int}(A)$, respectively, for $i = 1, 2$.

Definition 2.1. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(1) (i, j)-*semi-open* [11] if there exists a τ_i-open set U such that $U \subset A \subset j\mathrm{Cl}(U)$, equivalently if $A \subset j\mathrm{Cl}(i\mathrm{Int}(A))$, where $i \neq j$, i, $j = 1$, 2,

(2) (i, j)-*preopen* [4] if there exists a τ_i-open set U such that $A \subset U \subset j\mathrm{Cl}(A)$, equivalently if $A \subset i\mathrm{Int}(j\mathrm{Cl}(A))$, where $i \neq j$, i, $j = 1$, 2.

A subset A is said to be *pairwise semi-open* (resp. *preopen*) if it is $(1, 2)$-semi-open (resp. $(1, 2)$-preopen) and $(2, 1)$-semi-open (resp. $(2, 1)$-preopen). The complement of an (i,j)-semi-open (resp. (i,j)-preopen) set is said to be (i,j)-*semi-closed* (resp. (i,j)-*preclosed*). The (i,j)-*semi-closure* [11] (resp. (i,j)-*preclosure*) [9] of A, denoted by (i,j)-sCl(A) (resp. (i,j)-pCl(A)), is defined by the intersection of all (i,j)-semi-closed (resp. (i,j)-preclosed) sets containing A. The (i,j)-*semi-interior* [10] (resp. (i,j)-*preinterior*) of A, denoted by (i,j)-sInt(A) (resp. (i,j)-pInt(A)), is defined by the union of all (i,j)-semi-open (resp. (i,j)-preopen) sets contained in A.

Definition 2.2. A function $f : (X,\tau) \to (Y,\sigma)$ is said to be

(1) *weakly quasicontinuous* [15] (resp. *almost quasicontinuous* [14], *quasi-continuous* [12]) at a point x of X if for each open set G containing x and each open set V containing $f(x)$, there exists an open set U of X such that $\emptyset \neq U \subset G$ and $f(U) \subset \mathrm{Cl}(V)$ (resp. $f(U) \subset \mathrm{Int}(\mathrm{Cl}(V))$, $f(U) \subset V$),

(2) *weakly quasicontinuous* (resp. *almost quasicontinuous, quasicontinuous*) if it is weakly quasicontinuous (resp. almost quasicontinuous, quasicontinuous) at each point of X.

Lemma 2.1. *For a bitopological space (X,τ_1,τ_2), the following properties hold:*

(1) (i,j)-sCl$(A) = A \cup j\mathrm{Int}(i\mathrm{Cl}(A))$,

(2) $X - (i,j)$-sInt$(A) = (i,j)$-sCl$(X - A)$,

(3) *If a subset A of X is (i, j)-semi-open and $U \in \tau_1 \cap \tau_2$, then $A \cap U$ is (i, j)-semi-open,*

(4) $x \in (i,j)$-sCl(A) *if and only if $U \cap A \neq \emptyset$ for every (i,j)-semi-open set U containing x.*

Proof. This follows from Lemma 2.1 of [7], Theorem 1.13 of [10], Theorem 3 of [11] and Theorem 1.15 of [10]. □

Definition 2.3. A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *(i, j)-weakly semi-continuous* [7] at a point x of X if for each σ_i-open set V containing $f(x)$ there exists a (i,j)-semi-open set U of X containing x such that $f(U) \subset j\mathrm{Cl}(V)$.

A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *pairwise weakly semi-continuous* at a point $x \in X$ if it is (i,j)-weakly semi-continuous at $x \in X$ for $i,j = 1$, 2 and $i \neq j$.

Definition 2.4. A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *(i, j)-weakly quasicontinuous* [16] at a point x of X if for each σ_i-open set V containing $f(x)$ and each τ_j-open set G containing x, there exists a nonempty τ_i-open set U of X such that $U \subset G$ and $f(U) \subset j\mathrm{Cl}(V)$.

A function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be *pairwise weakly quasi-continuous* at $x \in X$ if it is (i,j)-weakly quasicontinuous at $x \in X$ for $i,j = 1$, 2 and $i \neq j$. A function f is said to be (i,j)-*weakly semi-continuous* (resp.

(i,j)-*weakly quasicontinuous*) if it is (i,j)-weakly semi-continuous (resp. (i,j)-weakly quasicontinuous) at each point of X.

In [16], it is shown that weak quasicontinuity is equivalent to weak semi-continuity in bitopological spaces.

3. CHARACTERIZATIONS OF WEAK QUASICONTINUITY

DEFINITION 3.1. Let (X, τ_1, τ_2) be a bitopological space and A a subset of X. A point x of X is said to be in (i,j)-*θ-closure* [5] of A, denoted by (i,j)-$\mathrm{Cl}_\theta(A)$, if $A \cap j\mathrm{Cl}(U) \neq \emptyset$ for every τ_i-open set U containing x, where $i, j = 1, 2$ and $i \neq j$.

A subset A of X is said to be (i,j)-*θ-closed* if $A = (i,j)$-$\mathrm{Cl}_\theta(A)$. A subset A of X is said to be (i,j)-*θ-open* if $X - A$ is (i,j)-θ-closed.

LEMMA 3.1. (Khedr [7], Popa and Noiri [16]) *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:*
(1) *f is (i,j)-weakly quasicontinuous;*
(2) $i\mathrm{Int}(j\mathrm{Cl}(f^{-1}(V))) \subset f^{-1}(j\mathrm{Cl}(V))$ *for each σ_i-open set V of Y;*
(3) $f^{-1}(V) \subset (i,j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)))$ *for every σ_i-open set V of Y;*
(4) (i,j)-$\mathrm{sCl}(f^{-1}(B)) \subset f^{-1}((i,j)$-$\mathrm{Cl}_\theta(B))$ *for every subset B of Y.*

THEOREM 3.1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following are equivalent:*
(1) *f is (i, j)-weakly quasicontinuous;*
(2) $i\mathrm{Int}(j\mathrm{Cl}(f^{-1}(V))) \subset f^{-1}(j\mathrm{Cl}(V))$ *for every (i,j)-preopen set V of Y;*
(3) (i,j)-$\mathrm{sCl}(f^{-1}(V)) \subset f^{-1}(i\mathrm{Cl}(V))$ *for every (j,i)-preopen set V of Y;*
(4) $f^{-1}(V) \subset (i,j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)))$ *for every (i,j)-preopen set V of Y.*

Proof. (1)⇒(2): Assume that f is (i,j)-weakly quasicontinuous. Let V be any (i,j)-preopen set of Y. Since f is (i,j)-weakly quasicontinuous, by Lemma 3.1 we obtain $i\mathrm{Int}(j\mathrm{Cl}(f^{-1}(V)) \subset i\mathrm{Int}(j\mathrm{Cl}(f^{-1}(i\mathrm{Int}(j\mathrm{Cl}(V))))) \subset f^{-1}(j\mathrm{Cl}(i\mathrm{Int}(j\mathrm{Cl}(V)))) \subset f^{-1}(j\mathrm{Cl}(V))$.

(2) ⇒ (3): Let V be any (j,i)-preopen set of Y. Then by (2) and Lemma 2.1 we have (i,j)-$\mathrm{sCl}(f^{-1}(V)) = f^{-1}(V) \cup j\mathrm{Int}(i\mathrm{Cl}(f^{-1}(V))) \subset f^{-1}(i\mathrm{Cl}(V))$.

(3) ⇒ (4): Let V be any (i,j)-preopen set of Y. Since every σ_j-open set is (j,i)-preopen, by (3) we have $X - (i,j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V))) = (i,j)$-$\mathrm{sCl}(X - f^{-1}(j\mathrm{Cl}(V)) = (i,j)$-$\mathrm{sCl}(f^{-1}(Y - j\mathrm{Cl}V))) \subset f^{-1}(i\mathrm{Cl}(Y - j\mathrm{Cl}(V))) = X - f^{-1}(i\mathrm{Int}(j\mathrm{Cl}(V))) \subset X - f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) \subset (i,j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)))$.

(4) ⇒ (1): Let V be any σ_i-open set of Y. Then V is (i,j)-preopen in Y and $f^{-1}(V) \subset (i,j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)))$. By Lemma 3.1, f is (i,j)-weakly quasicontinuous. □

DEFINITION 3.2. A bitopological space (X, τ_1, τ_2) is said to be (i,j)-*regular* [6] if for each $x \in X$ and each τ_i-open set U containing x, there exists a τ_i-open set V such that $x \in V \subset j\mathrm{Cl}(V) \subset U$.

LEMMA 3.2. *If a bitopological space* (X, τ_1, τ_2) *is* (i, j)*-regular, then* (i, j)-$\mathrm{Cl}_\theta(F) = F$ *for every* τ_i*-closed set* F.

Proof. Let $x \in (i, j)$-$\mathrm{Cl}_\theta(F)$, then $j\mathrm{Cl}(U) \cap F \neq \emptyset$ for every τ_i-open set U containing x. Since X is (i, j)-regular, there exists a τ_i-open set V such that $x \in V \subset j\mathrm{Cl}(V) \subset U$. Since $x \in (i, j)$-$\mathrm{Cl}_\theta(F), j\mathrm{Cl}(V) \cap F \neq \emptyset$. This implies that $U \cap F \neq \emptyset$ and hence $x \in i\mathrm{Cl}(F)$. Then we have $F \subset (i, j)$-$\mathrm{Cl}_\theta(F) \subset i\mathrm{Cl}(F) = F$. Therefore, we obtain (i, j)-$\mathrm{Cl}_\theta(F) = F$. □

DEFINITION 3.3. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-*semi-continuous* [11] (or (i, j)-*quasicontinuous*) at $x \in X$ if for each σ_i-open set V of Y containing $f(x)$, there exists an (i, j)-semi-open set U of X containing x such that $f(U) \subset V$.

A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be *pairwise semi-continuous* (or *pairwise quasicontinuous*) at $x \in X$ if it is (i, j)-semi-continuous (or (i, j)-quasicontinuous) at $x \in X$ and (j, i)-semi-continuous (or (j, i)-quasicontinuous) at $x \in X$. A function f is said to be (i, j)-*semi-continuous* (or (i, j)-*quasi-continuous*) if it is (i, j)-semi-continuous (or (i, j)-quasicontinuous) for each point $x \in X$.

THEOREM 3.2. *Let* (Y, σ_1, σ_2) *be an* (i, j)*-regular space. Then for a function* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following are equivalent:*
(1) f *is* (i, j)*-quasicontinuous;*
(2) $f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$ *is* (i, j)*-semi-closed for every subset* B *of* Y*;*
(3) f *is* (i, j)*-weakly quasicontinuous;*
(4) $f^{-1}(F)$ *is* (i, j)*-semi-closed for every* (i, j)*-*θ*-closed set* F *of* Y*;*
(5) $f^{-1}(V)$ *is* (i, j)*-semi-open for every* (i, j)*-*θ*-open set* V *of* Y*.*

Proof. (1) ⇒ (2): Since (i, j)-$\mathrm{Cl}_\theta(B)$ is σ_i-closed in Y, it follows from Theorem 16 of [11] that $f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$ is (i, j)-semi-closed in X.

(2) ⇒ (3): Let B be any subset of Y. Then by Proposition 3.8 of [3] and (2), we have (i, j)-$\mathrm{sCl}(f^{-1}(B)) \subset (i, j)$-$\mathrm{sCl}(f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B)))) = f^{-1}((i, j)$-$\mathrm{Cl}_\theta(B))$. By Lemma 3.1 f is (i, j)-weakly quasicontinuous.

(3) ⇒ (4): Let F be any (i, j)-θ-closed set of Y. By Lemma 3.1, (i, j)-$\mathrm{sCl}(f^{-1}(F)) \subset f^{-1}((i, j)$-$\mathrm{Cl}_\theta(F)) = f^{-1}(F)$. Therefore, it follows from Proposition 3.5 of [3] that $f^{-1}(F)$ is (i, j)-semi-closed.

(4) ⇒ (5): Let V be any (i, j)-θ-open set of Y. By (4), $f^{-1}(Y - V) = X - f^{-1}(V)$ is (i, j)-semi-closed in X and hence $f^{-1}(V)$ is (i, j)-semi-open.

(5) ⇒ (1): Since Y is (i, j)-regular, by Lemma 3.2, (i, j)-$\mathrm{Cl}_\theta(B) = B$ for every σ_i-closed set B of Y and thus every σ_i-open set is (i, j)-θ-open. Therefore, $f^{-1}(V)$ is (i, j)-semi-open for every σ_i-open set V of Y. Hence f is (i, j)-quasicontinuous. □

COROLLARY 3.1. (Popa and Noiri [15]) *Let* (Y, σ) *be a regular space. Then for a function* $f : (X, \tau) \to (Y, \sigma)$*, the following are equivalent:*
(1) f *is quasicontinuous;*
(2) $f^{-1}(\mathrm{Cl}_\theta(B))$ *is semi-closed for every subset* B *of* Y*;*

(3) *f is weakly quasicontinuous;*
(4) *$f^{-1}(F)$ is semi-closed for every θ-closed set F of Y;*
(5) *$f^{-1}(V)$ is semi-open for every θ-open set V of Y.*

4. WEAK QUASICONTINUITY AND QUASICONTINUITY

Definition 4.1. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i, j)-*weakly* quasicontinuous* (briefly (i, j)-w*.q.c.) if for every σ_i-open set V of Y, $f^{-1}(j\mathrm{Cl}(V) - V)$ is biclosed.

Theorem 4.1. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-weakly quasicontinuous and (i, j)-w*.q.c., then f is (i, j)-quasicontinuous.*

Proof. Let $x \in X$ and V be any σ_i-open set of Y containing $f(x)$. Since f is (i, j)-weakly quasicontinuous, there exists an (i, j)-semi-open set U of X containing x such that $f(U) \subset j\mathrm{Cl}(V)$. It is evident that $x \notin f^{-1}(j\mathrm{Cl}(V) - V)$. Therefore, $x \in U - f^{-1}(j\mathrm{Cl}(V) - V) = U \cap (X - f^{-1}(j\mathrm{Cl}(V) - V))$. Since U is (i, j)-semi-open and $X - f^{-1}(j\mathrm{Cl}(V) - V))$ is biopen, by Lemma 2.1 $G = U \cap (X - f^{-1}(j\mathrm{Cl}(V) - V))$ is (i, j)-semi-open. Then $x \in G$ and $f(G) \subset V$. Indeed, if $y \in G$, then $f(y) \notin j\mathrm{Cl}(V) - V$ and hence $f(y) \in V$. Therefore, f is (i, j)-quasicontinuous. □

Corollary 4.1. (Popa and Stan [17]) *If a function $f : X \to Y$ is weakly quasicontinuous and w*.c., then f is quasicontinuous.*

Definition 4.2. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to have the (i, j)-*sInteriority condition* if (i, j)-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V))) \subset f^{-1}(V)$ for each σ_i-open set V of Y.

Theorem 4.2. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i, j)-weakly quasicontinuous and satisfies the (i, j)-sInteriority condition, then f is (i, j)-quasicontinuous.*

Proof. Let V be any σ_i-open set of Y. Since f is (i, j)-weakly quasicontinuous, by Lemma 3.1, $f^{-1}(V) \subset (i, j)$-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)))$. By the (i, j)-sInteriority of f, we have (i, j)-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)) \subset f^{-1}(V)$ and (i, j)-$\mathrm{sInt}(f^{-1}(j\mathrm{Cl}(V)) \subset (i, j)$-$\mathrm{sInt}(f^{-1}(V)) \subset f^{-1}(V)$. Hence we obtain (i, j)-$\mathrm{sInt}(f^{-1}(V)) = f^{-1}(V)$. By Proposition 3.5 of [3], $f^{-1}(V)$ is (i, j)-semi-open in X and thus f is (i, j)-quasicontinuous. □

Definition 4.3. Let A be a subset of a bitopological space (X, τ_1, τ_2). The (i, j)-*semi-frontier* of A is defined as follows:

(i, j)-$\mathrm{sFr}(A) = (i, j)$-$\mathrm{sCl}(A) \cap (i, j)$-$\mathrm{sCl}(X - A) = (i, j)$-$\mathrm{sCl}(A) - (i, j)$-$\mathrm{sInt}(A)$.

Theorem 4.3. *The set of all points x of X at which $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is not (i, j)-weakly quasicontinuous is identical with the union of the (i, j)-semi-frontiers of the inverse images of the σ_j-closure of σ_i-open sets of Y containing $f(x)$.*

Proof. Let x be a point of X at which f is not (i,j)-weakly quasicontinuous. Then, there exists an σ_i-open set V of Y containing $f(x)$ such that $U \cap (X - f^{-1}(j\mathrm{Cl}(V))) \neq \emptyset$ for every (i,j)-semi-open set U of X containing x. By Lemma 2.1, $x \in (i,j)\text{-sCl}(X - f^{-1}(j\mathrm{Cl}(V)))$. Since $x \in f^{-1}(j\mathrm{Cl}(V))$, we have $x \in (i,j)\text{-sCl}(f^{-1}(j\mathrm{Cl}(V)))$ and hence $x \in (i,j)\text{-sFr}(f^{-1}(j\mathrm{Cl}(V)))$. Conversely, if f is (i,j)-weakly quasicontinuous, then for each σ_i-open set V of Y containing $f(x)$, there exists an (i,j)-semi-open set U of X containing x such that $f(U) \subset j\mathrm{Cl}(V)$ and hence $x \in U \subset f^{-1}(j\mathrm{Cl}(V))$. Therefore, we obtain that $x \in (i,j)\text{-sInt}(f^{-1}(j\mathrm{Cl}(V)))$. This contradicts that $x \in (i,j)$-sFr$(f^{-1}(j\mathrm{Cl}(V)))$. □

5. WEAK QUASICONTINUITY AND ALMOST QUASICONTINUITY

Definition 5.1. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-*almost quasicontinuous* [13] at $x \in X$ if for each τ_i-open set U of X containing x and each σ_i-open set V containing $f(x)$, there exists a nonempty τ_i-open set G of X such that $G \subset U$ and $f(G) \subset i\mathrm{Int}(j\mathrm{Cl}(V))$. The function f is said to be (i,j)-*almost quasicontinuous* if it has this property at each point $x \in X$.

Definition 5.2. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i,j)-*regular open* [18] if $A = i\mathrm{Int}(j\mathrm{Cl}(A))$.

Lemma 5.1. (Nasef [13]) *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is (i,j)-almost quasicontinuous;*

(2) *for each point $x \in X$ and each σ_i-open set V containing $f(x)$ there exists a (i,j)-semi-open set U of X containing x such that $f(U) \subset i\mathrm{Int}(j\mathrm{Cl}(V))$;*

(3) *$f^{-1}(V)$ is (i,j)-semi-open in X for each (i,j)-regular open set V of Y.*

Definition 5.3. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be (i,j)-*almost open* [8] if $f(U) \subset i\mathrm{Int}(j\mathrm{Cl}(f(U)))$ for every τ_i-open set U of X.

Theorem 5.1. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-weakly quasicontinuous and (i,j)-almost open, then f is almost quasicontinuous.*

Proof. Let $x \in X$ and V be a σ_i-open set of Y containing $f(x)$. There exists an (i,j)-semi-open set U of X containing x such that $f(U) \subset j\mathrm{Cl}(V)$. Since f is (i,j)-almost open, $f(U) \subset i\mathrm{Int}(j\mathrm{Cl}(f(U))) \subset i\mathrm{Int}(j\mathrm{Cl}(V))$. Therefore, by Lemma 5.1 f is (i,j)-almost quasicontinuous. □

Corollary 5.1. (Popa [14]) *If $f : X \to Y$ is an almost open and weakly quasicontinuous function, then f is almost quasicontinuous.*

Definition 5.4. A bitopological space (X, τ_1, τ_2) is said to be (i,j)-*almost regular* [18] if for each (i,j)-regular open set U of X containing x there exists an (i,j)-regular open set V of X such that $x \in V \subset j\mathrm{Cl}(V) \subset U$.

Theorem 5.2. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (i,j)-weakly quasicontinuous and (Y, σ_1, σ_2) is (i,j)-almost regular, then f is (i,j)-almost quasicontinuous.*

Proof. Let V be any (i,j)-regular open set of Y and x an arbitrary point in $f^{-1}(V)$. Then we have $f(x) \in V$. By the (i,j)-almost regularity of (Y, σ_1, σ_2), there exists an (i,j)-regular open set V_0 of Y such that $f(x) \in V_0 \subset j\mathrm{Cl}(V_0) \subset V$. Since f is (i,j)-weakly quasicontinuous, there exists an (i,j)-semi-open set U of X containing x such that $f(U) \subset j\mathrm{Cl}(V_0) \subset V$. This implies that $x \in U \subset f^{-1}(V)$. Therefore, we have $f^{-1}(V) \subset (i,j)$-sInt$(f^{-1}(V))$ and hence $f^{-1}(V) = (i,j)$-sInt$(f^{-1}(V))$. It follows from Proposition 3.5 of [3] that $f^{-1}(V)$ is (i,j)-semi-open and by Lemma 5.1 f is (i,j)-almost quasicontinuous. □

COROLLARY 5.2. (Popa [14]) *If a function $f : X \to Y$ is weakly quasicontinuous and Y is almost regular, then f is almost quasicontinuous.*

DEFINITION 5.5. A bitopological space (X, τ_1, τ_2) is said to be *pairwise Hausdorff* or *pairwise T_2* [6] if for each pair of distinct points x and y of X, there exist an τ_i-open set U containing x and a τ_j-open set V containing y such that $U \cap V = \emptyset$.

DEFINITION 5.6. A bitopological space (X, τ_1, τ_2) is said to be *pairwise semi-T_2* [7] if for each pair of distinct points x and y of X, there exist an (i,j)-semi-open set U containing x and a (j,i)-semi-open set V containing y such that $U \cap V = \emptyset$.

THEOREM 5.3. *Let (X, τ_1, τ_2) be a bitopological space. If, for each pair of distinct points x_1 and x_2 in X, there exists a function f of (X, τ_1, τ_2) into a pairwise T_2 bitopological space (Y, σ_1, σ_2) such that*

(a) *$f(x_1) \neq f(x_2)$,*

(b) *f is $(1,2)$-weakly quasicontinuous at x_1 and*

(c) *f is $(2,1)$-almost quasicontinuous at x_2, then the space (X, τ_1, τ_2) is pairwise semi-T_2.*

Proof. Let x_1 and x_2 be a pair of distinct points. Since (Y, σ_1, σ_2) is pairwise T_2, there exists a σ_i-open set V_i such that $f(x_i) \in V_i$ for $i = 1, 2$ and $V_1 \cap V_2 = \emptyset$. Since V_1 and V_2 are disjoint, we have σ_2-Cl$(V_1) \cap \sigma_2$-Int$(\sigma_1$-Cl$(V_2)) = \emptyset$. Since f is $(1,2)$-weakly quasicontinuous at x_1, there exists a $(1,2)$-semi-open set U_1 of X containing x_1 such that $f(U_1) \subset \sigma_2$-Cl(V_1). Since f is $(2,1)$-almost quasicontinuous at x_2, by Lemma 5.1 there exists a $(2,1)$-semi-open set U_2 of X containing x_2 such that $f(U_2) \subset \sigma_2$-Int$(\sigma_1$-Cl$(V_2))$. Hence we have $U_1 \cap U_2 = \emptyset$. This shows that (X, τ_1, τ_2) is pairwise-T_2. □

COROLLARY 5.3. (Popa [14]) *Let (X, τ) be a topological space. If, for each pair of distinct points x_1 and x_2 in X, there exists a function f of (X, τ) into a Hausdorff space (Y, σ) such that (a) $f(x_1) \neq f(x_2)$, (b) f is weakly quasicontinuous at x_1, and (c) f is almost quasicontinuous at x_2, then (X, τ) is semi-T_2.*

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