TWO-DIMENSIONAL POTENTIALS GENERATING A GIVEN
ONE-PARAMETER FAMILY OF ORBITS ON A SURFACE

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Abstract. We say that a potential generates a curve on a surface if a unit mass traces the curve under the action of the potential. We consider the following problem: a one-parameter family of regular curves \( f(u, v) = c \) on a surface \( r(u, v) = \{ x(u, v), y(u, v), z(u, v) \} \) is given. We seek two-dimensional potentials of the form \( V(u, v) = u^m R(u) \), \( R \) being an arbitrary \( C^2 \)-function, which generate this family of regular curves as trajectories on the above surface. We show that if the given family of orbits satisfies exactly two differential conditions, then such a potential exists and it is determined uniquely. Special cases are also studied and pertinent examples are given for each case. At a second step, if we consider that the “slope function” \( \gamma(u, v) = f_y/f_x \) is homogeneous of zero degree and the components of the metric tensor are homogeneous functions of zero degree too, then a potential of the above form always exists and it is found as a solution of an ordinary second-order O.D.E. Several examples are offered and implications of this study are discussed.

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REFERENCES


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