

SEMICONTINUITY AND CLOSEDNESS PROPERTIES  
OF RELATIONS IN RELATOR SPACES

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**Abstract.** We extend some basic theorems on semicontinuity and closedness properties of set-valued functions of topological spaces to relations on relator (generalized uniform) spaces.

Moreover, in addition, we prove that a closed-valued and uniformly lower semicontinuous relation on a topologically semisymmetric relator space to a uniformly topologically transitive relator space is closed.

**MSC 2000.** 54C60, 54E15.

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