

WILSON'S FUNCTIONAL EQUATION ON METABELIAN GROUPS

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Abstract. Consider the Wilson functional equation

$$f, g : G \rightarrow K, \quad f(xy) + f(xy^{-1}) = 2f(x)g(y)$$

where G is a group and K a quadratically closed field.

Aczél, Chung and Ng in 1989 have solved Wilson's equation, assuming that the function g satisfies Kannappan's condition $g(xyz) = g(xzy)$ and $f(xy) = f(yx)$ for all $x, y, z \in G$ and K is a quadratically closed field of $\text{char } K \neq 2$.

Investigations of Wilson's equation on non-abelian groups show that there exist solutions different of those obtained by Aczél, Chung and Ng.

In the present paper we obtain the general solution of Wilson's equation when G is a metabelian group all of whose commutators have finite order, and K a field with $\text{char } K = 0$ generalizing our result from [6] where this was obtained for P_3 -groups.

MSC 2000. Primary 39B52; Secondary 20B99.

Key words. Wilson's equation, metabelian group, P_3 -group.

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