A CONJECTURE (PARTLY) CONFIRMED BY COMPUTER

Lemma 1. *Let a, b, c, d be (collectively) coprime integers. There exist x, y, z, u such that $ax + by + cz + du = 1$ and $\det \begin{bmatrix} x & y \\ z & u \end{bmatrix} = 0$.*