An exercise in L. Fuchs's treatise

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In Fuchs, vol.2, p.239, we find

Ex.10 *List the groups* in which every endomorphism is either an automorphism or nilpotent.

In Fuchs 2015, p. 643, we find a "softer" version.

Ex. (2) *Find groups* in which every endomorphism is either an automorphism or nilpotent.

Maybe, in the 1973 version, this was too much for an exercise ?

First of all recall,

Lam **(19.3)**: (a) Suppose $R \neq 0$ and every $a \notin U(R)$ is nilpotent. Then R is a local ring.

Then recall,

(Orsatti): If End(G) is local then G is cocyclic or torsion-free.

A partial converse: the endomorphism ring of a cocyclic group is local, solves completely the torsion case.

So it only remains to discuss torsion-free groups with local endomorphism groups.

Related:

D. Arnold LN 931 (Finite rank t.f...), p. 25

Remark: It is proved in a later section that if A is strongly indecomposable then every endomorphism of A is a *monomorphism or else nilpotent.*

This is done at p. 92, the proof for (b).