Erratum

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In [1] **Example 4.7**, there are some typos (three signs and a 0 must be 1) in the fine decomposition.

The corrections are bold characters.

It should be $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 0 & \mathbf{1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & -\mathbf{1} \\ -\mathbf{1} & 0 & \mathbf{1} \\ 2 & -1 & 0 \end{bmatrix}$ (nilpotent + unit).

Another decomposition (P. Nielsen): $\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

Unfortunately there are also other typos. We thank Pace Nielsen for pointing these below.

[Ok means correction made].

It appears that throughout the paper you are implicitly assuming that rings are nonzero. This is needed in order for the definition of "fine ring" to be correct, and also for Proposition 2.3 to be correct. ok

* On page 233, line -18. (This is the next-to-last line of the proof of Theorem 2.4). Where it says $b \in nil(V)$, it should instead be $b \in nil(R)$. ok

* On page 236, in the proof of Proposition 3.9, you reference [6,Lemma 6.10]. In the cited proof of that result, they use $e_{1n} \neq 0$, but that is not necessarily true. Fortunately, I believe that this issue can be quickly fixed. ok

* In the statement of Lemma 4.2(2), b and c are only squares up to units.ok

* In the proof of Proposition 4.3, you say "Since $\gcd(x_1, y_1) = 1$ there exist $s, t \in T$ such that $sx_1 + ty_1 = 1$." This requires that R is a Bezout domain, which is a stronger assumption than R being a GCD domain. [Example: The ring $\mathbb{Z}[t]$ is a UFD, and hence a GCD domain. Moreover, $\gcd(2, t) = 1$ but 1 is not a linear combination of 2 and t.]ok

* In the proof of Theorem 4.4, the Bezout condition is again used (in the first sentence of the second paragraph).ok

* In the statement of Theorem 4.4, it is unclear what the plus-minus signs mean, since you already told the reader that equality is only defined up to units. If you meant for these to be true equalities, then they should instead

read $b = up^2$ and $q = -u^{-1}q^2$ for some unit $u \in R$. (By the way, this issue slightly complicates the proof, but not by much.)ok

* On page 239, line 6, both places that $B - T$ occur, it should be $B - T^2$.ok

* On page 240, on the third line of the second paragraph, c should be $-\mathrm{tr}(C)$ in both places.ok

* Three lines later, the square on $(\mathrm{tr} \begin{bmatrix} \mathrm{tr}(C) & \alpha \\ \beta & -\mathrm{tr}(C) \end{bmatrix})^2$ should be inside the parentheses.ok

Also, I believe that the $2\beta\alpha$ at the end of the line should just be $\beta\alpha$ (with no 2).ok

* Page 240, near the bottom. I believe that the columns of C^* and B^* were switched.ok

* Page 241. By "N-S corner" did you mean "N-W corner"?ok

By the way, in the proof of Proposition 3.9, I didn't see how to conclude the last sentence from the previous work. I found an alternative argument, but it was somewhat complicated.

Answer. For the last sentence of the proof of Prop 3.9, every idempotent of T is central because of [6, Lemma 6.10], we should have written: for any nonzero idemptent t of T, $e = tI_n$ is an idempotent of R such that $eRe = \mathbb{M}_n(tTt)$, so e is central by [6, Lemma 6.10], and hence t is central.

A corrected version is attached on my Homepage.

References

[1] G. Călugăreanu, Y. Zhou *Rings with fine nilpotents*. Annali del Univ. Ferrara **67** (2) (2021), 231-241.