Ab5* abelian groups

Definition 1. A subset D of a poset A is called *upper (lower) directed* if each finite (two-element) subset of D has an upper (respectively lower) bound in D. A lower directed family of submodules of a module is called (see [2]) an *inverse family of submodules*.

Definition 2. A module M is *Ab5** if for every submodule $K \subset M$ and every inverse family $\{N_i\}_{i \in I}$ of submodules of M,

$$K + \bigcap_I N_i = \bigcap_I (K + N_i).$$

Definition 3. A complete lattice L is called *lower continuous* if $a \vee (\bigwedge D) = \bigwedge_{d \in D} (a \vee d)$ holds for every $a \in L$ and every lower directed subset $D \subseteq L$.

Hence, *a module M is Ab5* iff the submodule lattice $L(M)$ is lower continuous.* Rephrasing we obtain

Theorem. ([1]) *An Abelian group is Ab5* iff it is torsion with Artinian (i.e., finitely cogenerated = finite direct sums of cocyclic groups) primary components.*

References

[1] Benabdallah K., Călugăreanu G., *Abelian groups with continuous lattice of subgroups.* Studia Univ. Babeş-Bolyai Math. 32 (1987), no. 1, 31–32.

[2] Wisbauer R., *Foundations of Module and Ring Theory*, Gordon and Breach Science Publishers, 1991.