Abstract homotopy theory for cyclic operads

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Abstract

Cyclic operads were first introduced in the category of chain complexes by Getzler and Kapranov, who used these structures to give a unifying view on the cyclic homology for various kinds of algebras. It turns out that all these algebras can be viewed as algebras over some cyclic operads, and the extra cyclic structure of the operad contains all the information needed to define cyclic homology for these algebras and to derive a generalised version of Connes' SBI-sequence to compute cyclic homology.

Cyclic operads were anticipated also by a theorem of Kontsevich relating the homology of certain infinite dimensional Lie algebras to graph homology. The ideas of Kontsevich have been extensively studied by Conant and Vogtmann, this time with the explicit use of cyclic operads.

A natural question that arises is whether the abstract homotopy theory of operads, as well as the general W-construction provided by Berger and Moerdijk's work can be extended to cyclic operads.

The aim of this talk is to prove the existence of a model structure on the category of cyclic operads in a symmetric monoidal model category, under certain assumptions. A model structure is obtained in two separate cases. In the first case we use a transfer principle to transport the model structure from the category of cyclic collections. In the second case we consider unreduced cyclic operads. To obtain a model structure in this case, we construct various algebras over coloured operads, and then we can use a theorem of Berger and Moerdijk to establish the desired model structure.

The model structures then are used to obtain a Boardman-Vogt type cofibrant resolution of cyclic operads in symmetric monoidal categories. In the case of differential graded vector spaces this cofibrant resolution can be considered as an extension of the classical bar-cobar resolution of operads to the cyclic case.