Group graded bimodules over commutative G-rings

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Let G be a finite group, Z a commutative G-ring, and let $F = Z^G$. Let A and B be two G-acted F-algebras such that $Z \to Z(A)$ and $Z \to Z(A)$ are G-ring homomorphisms. Then the tensor product over Z of A and B is again a G-acted F-algebra over Z. Motivated by the study of Clifford theory in combination with Schur indices, A. Turull has introduced an equivalence relation between simple algebras of this kind, which comes down to the notion of equivariant Morita equivalence over Z between them. However, strongly graded algebras are natural for Clifford theory. So let R and S be two strongly G-graded algebras such that $Z \to Z(R_1)$ and $Z \to Z(S_1)$ are G-ring homomorphisms.

Turull's equivalence classes over F (defined by A Turull in 1994) can be generalized to the case of strongly G-graded algebras (see A. Marcus (2008, 2009). The problem is that the tensor product over Z of R and S is no longer an algebra. Nevertheless, we show that we can still consider G-graded Morita equivalences over Z (not only over F) between R and S and we discuss the particular case of skew-group algebras and the relation between G-graded Morita equivalences and G-equivariant Morita equivalences.

Finally, we associate a central simple G-graded algebra over Z to a character of R, and we compare this with Turull's central simple G-algebra associated to that character.