International conference on
MODULES AND REPRESENTATION THEORY
Cluj-Napoca, July 7 – 12, 2008

The roots of Ext

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This talk will survey recent set– and model–theoretic algebra tools for making Ext vanish.

If $\mathcal{C}$ is a class of modules then the modules M such that $\mathrm{Ext}^1(M,C) = 0$ for each $C \in \mathcal{C}$ are called the *roots of Ext for* $\mathcal{C}$. The roots are ubiquitous in module and representation theory: they include the projective, flat, and Baer modules, for example, and they are closely related to tilting and cotilting modules.

A basic set–theoretic technique of the structure theory of the roots consists in building filtrations whose consecutive factors are small roots of the same kind. This is the *deconstruction* of the roots. For example, the classical Kaplansky structure theorem for projective modules is just a simple case of the deconstruction when $\mathcal{C}$ is the class of all modules.

In the first part of my talk, I will explain how set–theoretic methods yield deconstruction with countably, and even finitely generated root factors. I will also give some applications, [1], [3], [4], [5].

The second part will present a new link to model theory: the roots of Ext yield natural examples of abstract elementary classes (AEC's) in the sense of Shelah. I will finish by a recent result saying that the AEC's of the roots induced by tilting and cotilting modules are always of finite character in the sense of Hyttinen and Kesälä; [2], [6].

References

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