t-structures in the bounded derived category of a commutative Noetherian ring

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The concept of t-structure in a triangulated category is the analogue in the 'triangulated world' of what a (hereditary) torsion pair is in the 'abelian world'. The problem of classifying the t-structures in the bounded derived category of coherent sheaves on an affine space $\mathbf{X} = Spec(R)$ is open. We shall approach the Noetherian case, by showing that if R is a commutative Noetherian ring then we can assign to every t-structure in $D^b_{fg}(R)$ a uniquely determined decreasing filtration $\phi : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$ satisfying the following two conditions:

1. $\phi(i)$ is stable under specialization, for every $i \in \mathbf{Z}$

2. ϕ satisfies the weak Cousin condition: if $\mathbf{p} \subset \mathbf{q}$ is an inclusion of prime ideals, with $\mathbf{p}$ maximal under $\mathbf{q}$ and $\mathbf{q} \in \phi(j)$, then $\mathbf{p} \in \phi(j-1)$

We will finally show that if R has a dualizing complex, then the above assignment is exhaustive and, hence, establishes a bijection between t-structures in $D^b_{fg}(R)$ (equivalently in $D^b(coh\mathbf{X})$ for $\mathbf{X} = Spec(R)$) and filtrations $\phi : \mathbf{Z} \longrightarrow \mathcal{P}(Spec(R))$ satisfying properties 1 and 2.