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Relation between Lie and Jordan structures

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Given a 3-graded Lie algebra $L = L_{-1} \oplus L_0 \oplus L_1$, the formula $\{x, y, z\} = [[x, y], z]$ defines a Jordan pair structure on (L_1, L_{-1}) as soon as $\frac{1}{6} \in \Phi$ (cf. [8, 2.2(b); 9, 1.2]). Conversely, from a Jordan pair $V = (V^+, V^-)$ we can always build a 3-graded Lie algebra through the Tits-Kantor-Koecher construction [9, §1]. This type of Lie algebras was first considered by Tits [10], Kantor [3; 4; 5] and Koecher [6; 7], and gives a closed relation between Lie and Jordan structures that has been very fruitful.

In [1; 2] we attach a Jordan algebra L_x to any ad-nilpotent element x of index of nilpotence at most 3 in a Lie algebra L, and a Jordan pair $V = (B, L/Ker(B))$ to every abelian inner ideal B of L. These Jordan systems behave similarly to local algebras and subquotients of Jordan systems, and have made possible to study, and even to classify, infinite dimensional Lie algebras with chain conditions on abelian inner ideals.

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